

Building the ICSO

Development and Validation of the Interior Constitutive State Operator Solver



A step-by-step account of the construction, testing, and validation of the computational solver used to investigate the Interior Accumulation Web.

This section documents how the numerical implementation of the Interior Constitutive State Operator (ICSO) emerged through a sequence of progressively more demanding simulations, leading to a fully qualified, frozen, and locked solver architecture.

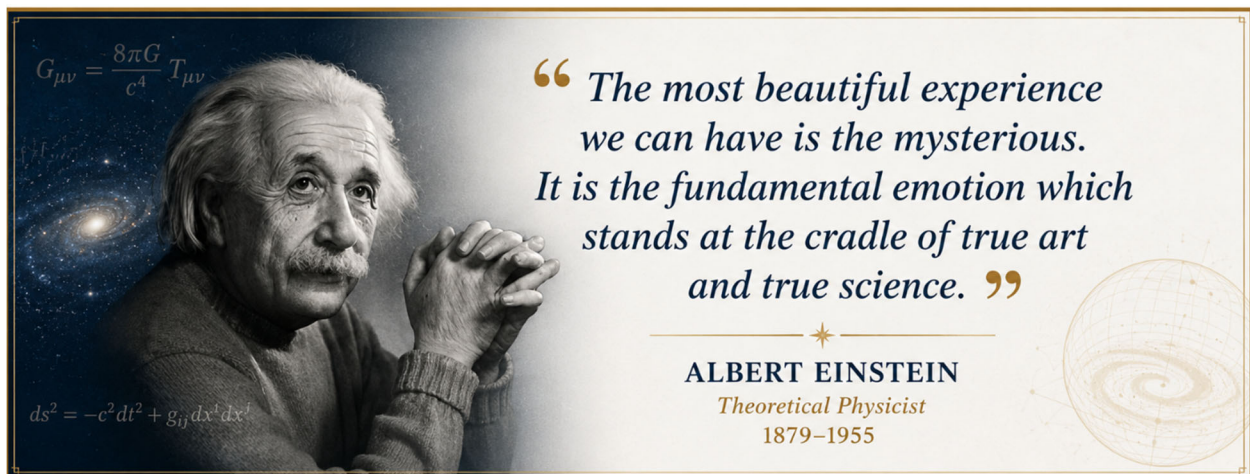


Energy is neither created nor destroyed

The law of conservation of energy says that energy is neither created nor destroyed. When people use energy, it doesn't disappear, but instead, it changes from one form of energy into another form.

“Energy changes form, but the total amount of energy in the universe stays the same.”

First Law of Thermodynamics



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Building the ICSO

From Mathematical Architecture to a Validated Computational Solver

The Interior Accumulation Web required more than a governing field equation. If the proposed constitutive framework was going to be investigated computationally, it required a numerical system capable of evolving the structural field without being programmed to produce a preferred physical outcome. That requirement became the foundation for the development of the **Interior Constitutive State Operator (ICSO)** and the numerical solver used to investigate its associated structural dynamics.

The development did not begin with a finished computational architecture. It proceeded through a sequence of simulations, diagnostic tests, failures, corrections, and increasingly demanding validation exercises. Each stage was allowed to expose weaknesses in the previous stage. Numerical methods were retained only when they survived those tests. This distinction is important. The purpose of the Simulation 20 series was not to demonstrate that the IAW hypothesis was correct. Its purpose was to determine whether the mathematical system being constructed to investigate the hypothesis could be trusted to calculate its own evolution. The governing development principle became:

Do not program the expected endpoint.

Instead,

Program the physics and calculate the endpoint.

This principle separated the numerical machinery from the physical conclusions that machinery might eventually produce.

Why the ICSO Was Necessary

The IAW begins with a relativistic physical state. That state alone does not determine the constitutive response proposed by the hypothesis.

A mathematical bridge was therefore required between the relativistic description of the interior and the effective constitutive quantities governing the IAW structural field.

That bridge became the **Interior Constitutive State Operator**:

$$\mathcal{I}: \mathcal{S}_{\text{rel}} \rightarrow \mathcal{C}_{\text{IAW}}$$

where the constitutive state is represented by

$$\mathcal{C}_{\text{IAW}} = \{\rho_{\text{eff}}, \mathbf{K}^{ij}, \mu^2, \lambda, \gamma, S\}.$$

The resulting architecture developed into the sequence

$$\text{Relativistic State} \rightarrow \text{Observer Projection} \rightarrow \text{ICSO} \rightarrow \text{Constitutive State} \rightarrow \text{Structural Field Dynamics}.$$

The structural field is represented by

$$\Psi = \Psi_1 + \Psi_2,$$

with Ψ_2 representing the persistent or background IAW constitutive medium and Ψ_1 representing localized constitutive organization developing within that medium. Importantly,

$$\Psi_2 \neq 1 - \Psi_1.$$

The two components are not defined as complementary fractions of a normalized whole. This distinction would later become important when earlier computational assumptions were subjected to audit.

From Mathematical Definition to Numerical Experiment

The governing structural-field equation developed into the form

$$\partial_t(\rho_{\text{eff}} \partial_t \Psi) - \nabla_i (K^{ij} \nabla_j \Psi) + \mu^2 \Psi + \lambda \Psi^3 + \gamma \partial_t \Psi = S.$$

This equation contains several physically distinct behaviors:

- structural propagation,
- effective inertia,

- linear restoring response,
- nonlinear self-interaction,
- dissipation,
- and external or internal forcing.

Writing such an equation is only the beginning. A computational implementation must demonstrate that it solves the intended mathematical system, remains numerically stable, conserves or dissipates energy according to the equation being integrated, converges as resolution increases, and continues to behave coherently when its parameters are moved away from a convenient baseline. The Simulation 20 sequence was constructed to perform that work.

The Simulation 20 Development Program

The Simulation 20 series became a staged qualification program. Each simulation asked a progressively more difficult question of computational architecture. The sequence did not assume that the existing solver was correct. A simulation was allowed to fail. A diagnostic was allowed to reveal an inconsistency. An earlier assumption was allowed to be rejected.

This produced an important methodological rule:

A failed test is information, not a result to be hidden.

When a numerical or architectural problem was discovered, the appropriate response was not to loosen the acceptance criterion until the simulation passed. The problem was identified, isolated, and corrected before the next stage proceeded. Consequently, the final solver was not simply the original computational model with additional features attached to it. It was the result of a progressive elimination of assumptions and numerical practices that could not survive validation.

Simulation 20.1 — Establishing the Computational Foundation

Objective

Simulation 20.1 began the transition from the mathematical IAW framework into a computational system capable of evolving a structural field through time. At this stage, the central problem was fundamental: Can the proposed structural dynamics be represented as a numerically evolvable field system without prescribing the solution that the system is expected to reach?

The initial simulation architecture established the basic numerical environment from which the later Simulation 20 series would develop. The field had to possess a defined spatial domain, an initial state, an evolution rule, and a method for calculating successive states through time. The computational problem can be represented schematically as

$$[\Psi, (\mathbf{x}, t_0) \partial_t \Psi(\mathbf{x}, t_0)] \rightarrow \text{Numerical Evolution} \rightarrow [\Psi, (\mathbf{x}, t) \partial_t \Psi(\mathbf{x}, t)].$$

The crucial requirement was that the state appearing on the right side of this expression could not be predetermined. The solver had to calculate it.

Development Principle Established

Simulation 20.1 therefore established the first major rule of the Simulation 20 program:

Initial conditions are inputs. Final conditions are outputs.

A numerical experiment could specify the initial field configuration and the coefficients governing its evolution. It could not specify what the final field should become and then construct the numerical system around that answer. This became the computational foundation for every subsequent Simulation 20 test.

What Simulation 20.1 Did Not Establish

Simulation 20.1 was not intended to validate the entire IAW computational framework.

It did not establish physical IAW parameter values. It did not identify a fundamental IAW length scale. It did not establish that the constitutive interpretation of every calculated quantity was correct. And it did not demonstrate that the solver would remain reliable under increasing numerical or physical stress. Those questions required separate tests.

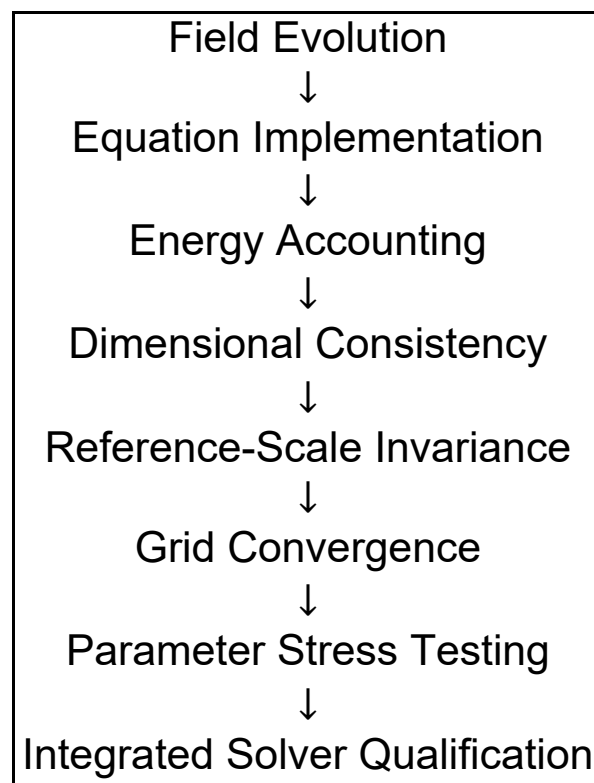
This distinction became increasingly important as the Simulation 20 sequence progressed:

Successful execution $\neq$ numerical validation.

A program can run without crashing and still calculate the wrong answer. The remaining Simulation 20 series would therefore progressively attack the solver itself.

The Validation Ladder

The development path that follows can be viewed as a validation ladder:



No single simulation was expected to establish all of these properties. Instead, each stage was designed to remove another source of numerical ambiguity. By the end of the sequence, the question would no longer be merely:

Does the program run?

The question would become: **Has the numerical engine earned sufficient confidence to be used for the next stage of IAW investigation?**

That is the question Simulation 20.10 answered.

The energy plot explains the error plot

The kinetic and gradient energies are exchanging periodically while their sum stays visually almost constant. That's precisely what should happen:

$$E = E_K + E_G.$$

When the Gaussian is momentarily reconstructed, gradient energy dominates. During propagation, energy transfers between gradient and kinetic components. I inspected the history data that I uploaded more closely.

At

$$t = 0.25:$$

$$E_K \approx 2.769327 \times 10^{-3}$$

and

$$E_G \approx 2.769 \times 10^{-3}.$$

So, the energy is exactly split 50/50.

At

$$t = 0.5:$$

kinetic energy falls completely away:

$$E_K \approx 1.84 \times 10^{-9},$$

while the gradient energy returns to the original total.

At

$$t = 1.0:$$

The same reconstruction occurs again. That is a very clean oscillatory energy exchange.

The energy-error oscillation is synchronized with the wave cycle

This is an important finding. At the reconstructed states:

$$t = 0.5$$

the relative energy error is only approximately

$$3.99 \times 10^{-11},$$

and at

$$t = 1.0$$

it is approximately

$$2.05 \times 10^{-10}.$$

Yet between reconstructed states it rises. That strongly supports the discrete-energy-diagnostic explanation. The solver is not simply gaining or losing 1.2×10^{-4} energy every cycle. If it were, it would not repeatedly return extremely close to the original energy. Instead, we're seeing

periodic diagnostic excursion + very small long-term numerical drift.

Those are two different effects.

And the convergence result is stronger than it initially looked

The maximum errors were:

$$2.861275 \times 10^{-4},$$

$$1.154287 \times 10^{-4},$$

$$2.884809 \times 10^{-5}.$$

The last refinement ratio is

$$\frac{1.154287 \times 10^{-4}}{2.884809 \times 10^{-5}} \approx 4.0013.$$

Therefore, the measured convergence order is

$$p = \log_2(4.0013) \approx \boxed{2.00045}.$$

That's the exact second-order convergence between $N = 1024$ and $N = 2048$. Even more interesting are the final errors after one full crossing:

N	Final relative energy error
512	2.48×10^{-8}
1024	2.05×10^{-10}
2048	2.50×10^{-12}

When the pulse returns to its initial configuration, the numerical solution is recovering the original energy extraordinarily closely. That reinforces the conclusion. I still want the D2 test due to the results of the last output. Currently diagnosed

$$E_G^{(D1)} = \frac{1}{2} c_\chi^2 \sum_j (D_1 \chi_j)^2 \Delta x.$$

But the actual equation has evolved with

$$D_2 \chi.$$

For the periodic centered Laplacian, define the corresponding discrete potential energy as:

$$E_G^{(D2)} = -\frac{1}{2}c_\chi^2 \sum_j \chi_j (D_2 \chi)_j \Delta x.$$

Then track both simultaneously:

$$E_{\text{total}}^{(D1)} = E_K + E_G^{(D1)}$$

and

$$E_{\text{total}}^{(D2)} = E_K + E_G^{(D2)}.$$

If the diagnosis is correct, I expect the large periodic teeth in the third figure to collapse dramatically for the $D2$ -consistent energy. That would be a very satisfying audit result because I would have identified the numerical reason, rather than merely accepting that the error is small. No physics change. This does not alter:

$$\chi_{tt} = c_\chi^2 \chi_{xx}.$$

It does not change the Gaussian, RK4, grid, timestep, boundary condition, or derivative operator. I'm simply putting a second measuring instrument on the same simulation.

20.1A — Discrete Energy Diagnostic Audit

Then I ran the identical 20.1 solver with D1 and D2 energy calculations side-by-side. D2 behaved as expected, I documented the findings, selected the mathematically consistent diagnostic, and then:

LOCKED SIMULATION 20.1

After that, 20.2 gets exactly one new physical component.

Figure 01. Simulation 20.1 — Initial Computational Architecture. The first stage of the Simulation 20 program established the numerical evolution framework from which the subsequent ICSSO solver architecture was developed. Initial conditions and governing coefficients were supplied to the solver, while the resulting field evolution remained a calculated output.

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IAW SIMULATION 20.1 — CLEAN DIMENSIONLESS WAVE BASELINE

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Solver Mode : Dimensionless

Reference Scales : Not assigned

Physics : $\chi_{tt} = c_{\chi}^2 \chi_{xx}$

Boundary : Periodic derivative operator only

N : 1024

L : 1.0000000000

dx : 9.7656250000e-04

dt : 5.0000000000e-04

T_final : 80.000000

c_chi : 1.000000

CFL : 0.512000

Gaussian A0 : 0.025000

Gaussian x0 : 0.500000

Gaussian sigma : 0.050000

Initial velocity : 0

Initial energy : 5.537861953582e-03

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Step 10000 t= 5.0000 E=5.5378620223e-03 rel.energy.error=1.240e-08

Step 20000 t= 10.0000 E=5.5378622483e-03 rel.energy.error=5.322e-08

Step 30000 t= 15.0000 E=5.5378626310e-03 rel.energy.error=1.223e-07

Step 40000	t= 20.0000	E=5.5378631694e-03	rel.energy.error=2.195e-07
Step 50000	t= 25.0000	E=5.5378638619e-03	rel.energy.error=3.446e-07
Step 60000	t= 30.0000	E=5.5378647068e-03	rel.energy.error=4.972e-07
Step 70000	t= 35.0000	E=5.5378657017e-03	rel.energy.error=6.768e-07
Step 80000	t= 40.0000	E=5.5378668441e-03	rel.energy.error=8.831e-07
Step 90000	t= 45.0000	E=5.5378681309e-03	rel.energy.error=1.115e-06
Step 100000	t= 50.0000	E=5.5378695590e-03	rel.energy.error=1.373e-06
Step 110000	t= 55.0000	E=5.5378711247e-03	rel.energy.error=1.656e-06
Step 120000	t= 60.0000	E=5.5378728243e-03	rel.energy.error=1.963e-06
Step 130000	t= 65.0000	E=5.5378746535e-03	rel.energy.error=2.293e-06
Step 140000	t= 70.0000	E=5.5378766082e-03	rel.energy.error=2.646e-06
Step 150000	t= 75.0000	E=5.5378786838e-03	rel.energy.error=3.021e-06
Step 160000	t= 80.0000	E=5.5378808756e-03	rel.energy.error=3.417e-06

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SIMULATION 20.1 AUDIT SUMMARY

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CFL test : PASS

Finite-solution test : PASS

Initial total energy : 5.537861953582e-03

Final total energy : 5.537880875642e-03

Final relative energy error : 3.416852949056e-06

Max relative energy error : 1.208720020880e-04

Grid convergence:

N= 512 dx=1.953125e-03 dt=1.000000e-03 CFL=0.512000 max dE/E=2.861275e-04

N=1024 dx=9.765625e-04 dt=5.000000e-04 CFL=0.512000 max dE/E=1.154287e-04

N=2048 dx=4.882812e-04 dt=2.500000e-04 CFL=0.512000 max dE/E=2.884809e-05

NOTE: No arbitrary energy-error pass threshold is imposed in 20.1.

The measured error and convergence behavior are reported for audit.

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Simulation 20.1A — Discrete Energy Diagnostic Audit

Simulation 20.1 established the clean dimensionless wave baseline. Its governing equation was intentionally minimal:

$$\chi_{tt} = c_\chi^2 \chi_{xx}$$

with no restoring term, nonlinear interaction, damping, or external forcing. The baseline evolved successfully, but successful field evolution alone was not sufficient. Before adding another physical term, the numerical energy diagnostic itself needed to be examined. This led directly to Simulation 20.1A — Discrete Energy Diagnostic Audit. Unlike the simulations that followed, 20.1A did not change the governing physics.

The Simulation 20.1A record states explicitly: “20.1A changes diagnostics only; the 20.1 PDE and integrator are unchanged.”

This distinction became important to the entire validation strategy. A problem discovered in a diagnostic should not automatically be interpreted as a failure of the physical equation or numerical integrator. Sometimes the measuring instrument is what must be examined.

Why 20.1A Was Necessary

For the conservative wave equation,

$$\chi_{tt} = c_\chi^2 \chi_{xx},$$

The continuous energy has the familiar form

$$E = \frac{1}{2} \int [\chi_t^2 + c_\chi^2 (\partial_x \chi)^2] dx.$$

If the numerical implementation accurately represents this conservative system, its calculated energy should remain nearly constant, with deviations decreasing as numerical resolution improves. Simulation 20.1 had already provided an energy history. Simulation 20.1A asked a deeper question:

Is the discrete energy diagnostic itself consistent with the discrete spatial operator used by the solver? That question required no new IAW physics. It required an audit of how the existing numerical solution was being measured.

Physics Was Held Fixed

Simulation **20.1A** retained exactly the wave equation used in **20.1**:

$$\chi_{tt} = c_{\chi}^2 \chi_{xx}.$$

The simulation remained dimensionless, and no reference scales were assigned. The principal configuration was:

$$N = 1024, L = 1,$$

$$\Delta x = 9.765625 \times 10^{-4},$$

$$\Delta t = 5.0 \times 10^{-4},$$

$$T_{\text{final}} = 80,$$

and

$$c_{\chi} = 1.$$

The Courant number remained

$$\text{CFL} = 0.512.$$

The initial condition also remained the same Gaussian disturbance:

$$A_0 = 0.025, x_0 = 0.5, \sigma = 0.05,$$

with

$$\chi_t(x, 0) = 0.$$

Nothing in this configuration attempted to improve the result by altering the physical system. The variable under examination was the energy diagnostic.

Two Discrete Energy Measurements

Simulation 20.1A evaluated the same field evolution using two discrete energy representations, designated D1 and D2. Their initial calculated energies were

$$E_{D1}(0) = 5.537861953582 \times 10^{-3}$$

and

$$E_{D2}(0) = 5.538654176270 \times 10^{-3}.$$

The difference between these initial values was small, but the purpose of the test was not simply to compare their starting magnitudes. The important question was how each diagnostic behaved as the field evolved through **160,000 time steps**.

At

$$t = 80,$$

D1 produced

$$E_{D1}(80) = 5.537880875642 \times 10^{-3},$$

corresponding to a final relative energy error of

$$\boxed{3.416852949056 \times 10^{-6}}.$$

Its maximum measured relative error during the audit reached

$$\boxed{1.208720020880 \times 10^{-4}}.$$

D2 produced

$$E_{D2}(80) = 5.538654014820 \times 10^{-3},$$

with a final—and maximum—relative error of only

$$\boxed{2.914973053392 \times 10^{-8}.$$

The same evolving field therefore appeared different depending upon how its discrete energy was evaluated. That was a major numerical finding.

The Diagnostic Was Now Being Tested, Not Assumed

At this point in the development, it would have been easy to select the diagnostic producing the smaller error and simply proceed. That would not constitute validation. Instead, the diagnostics were subjected to a grid-refinement test. The purpose was to determine whether the measured energy error behaved systematically as the numerical representation was refined. Three grids were tested:

$$N = 512, 1024, 2048.$$

The timestep was reduced proportionally with the grid spacing so that

$$\boxed{\text{CFL} = 0.512}$$

remained unchanged across all three calculations. The resulting configurations were:

N	Δx	Δt	CFL
512	1.953125×10^{-3}	1.0×10^{-3}	0.512
1024	9.765625×10^{-4}	5.0×10^{-4}	0.512
2048	4.882812×10^{-4}	2.5×10^{-4}	0.512

This is important because resolution could now be changed without simultaneously changing the Courant condition.

Grid Refinement Exposed the Difference

The D1 maximum relative energy errors were

$$2.861275 \times 10^{-4}$$

for $N = 512$,

$$1.154287 \times 10^{-4}$$

for $N = 1024$,

and

$$2.884809 \times 10^{-5}$$

for $N = 2048$.

Thus,

$$2.861275 \times 10^{-4} \rightarrow 1.154287 \times 10^{-4} \rightarrow 2.884809 \times 10^{-5}.$$

The D1 error decreased as the grid was refined. D2 exhibited an even stronger progression:

$$1.163960 \times 10^{-8}$$

at $N = 512$,

$$3.643690 \times 10^{-10}$$

at $N = 1024$,

and

$$1.138828 \times 10^{-11}$$

at $N = 2048$.

Therefore,

$$\boxed{1.163960 \times 10^{-8} \rightarrow 3.643690 \times 10^{-10} \rightarrow 1.138828 \times 10^{-11}.}$$

This was not random numerical behavior. As resolutions increased, the measured error systematically collapsed.

What 20.1A Changed

The most important aspect of Simulation 20.1A is what did not change. It did not alter

$$\chi_{tt} = c_\chi^2 \chi_{xx}.$$

It did not modify the time integrator. It did not add a physical coefficient. It did not change the initial field to obtain a better result. It did not assign a physical IAW length, time, or field scale. Instead:

20.1 Physics + 20.1 Integrator + Revised Diagnostic Audit

was used to determine whether the apparent energy behavior originated in the evolving field or in the method used to evaluate its discrete energy. The execution record makes this separation explicit:

20.1A changes diagnostics only; the 20.1 PDE and integrator are unchanged.

This became an important precedent for later simulations. When a validation test failed, the development process would first determine what had actually failed before changing the solver.

Numerical Result

The formal audit produced:

CFL Test: PASS

and

Finite-Solution Test: PASS.

The field remained finite through the full simulation interval, and the diagnostic study demonstrated systematic improvement under grid refinement. The D2 energy diagnostic in particular reached a maximum relative error of

$$2.914973053392 \times 10^{-8}$$

during the principal $N = 1024$, $T = 80$ run, while the $N = 2048$ grid-refinement calculation reduced its maximum error to

$$1.138828 \times 10^{-11}.$$

These results established a sufficiently controlled conservative baseline to proceed to the first additional physical term.

Why 20.1A Matters to the ICSO Development

Simulation 20.1A appears, at first glance, to be a small correction between larger simulations. It was not. It established a methodological principle that would become increasingly important as the solver became more complicated:

The solver and the diagnostic used to judge the solver are separate numerical ob

A diagnostic must therefore earn confidence just as the field evolution must earn confidence. That principle would later become critical when damping, forcing, compensated energy balance, scale transformations, and convergence testing were introduced. The development process had now established two separate requirements:

**Does the numerical field evolve correctly?
and
Are we measuring that evolution correctly?**

Simulation 20.1A addressed the second question before additional physics was allowed into the solver.

Simulation 20.1A Disposition

The governing PDE and integrator from Simulation 20.1 remained unchanged. The CFL and finite-solution tests passed, and the energy diagnostic demonstrated controlled grid-refinement behavior.

Simulation 20.1A therefore became the frozen baseline inherited by Simulation 20.2. The subsequent 20.2 execution record explicitly identifies its starting point as:

Baseline: Simulation 20.1A LOCKED.

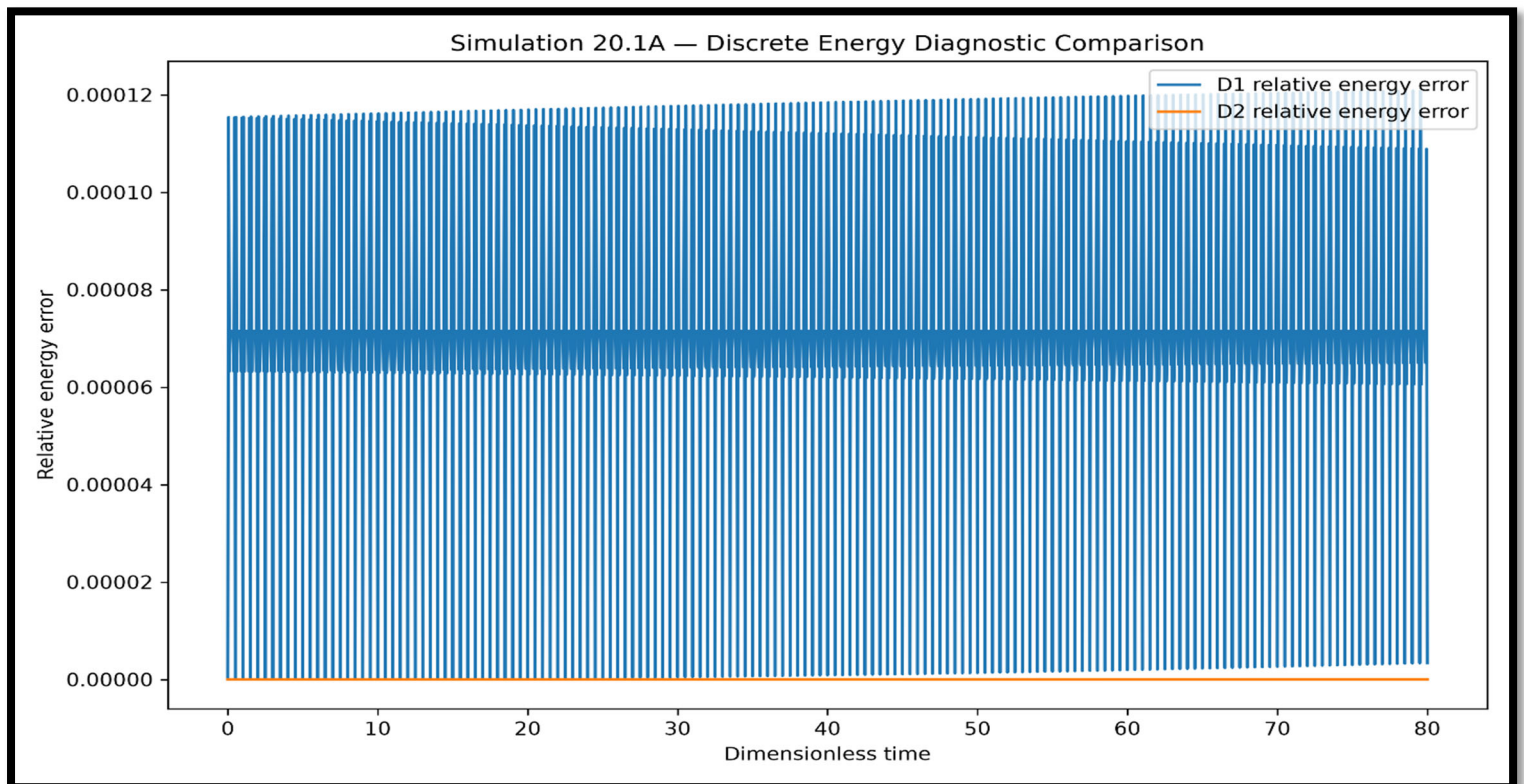
Thus the development chain became

20.1 → 20.1A Diagnostic Audit → 20.1A LOCKED → 20.2.

Only after that diagnostic foundation was established was the first new physical term introduced. That term was the linear restoring response,

$$-\mu_{\chi}\chi,$$

which became the subject of Simulation 20.2 — Linear Restoring-Term Audit



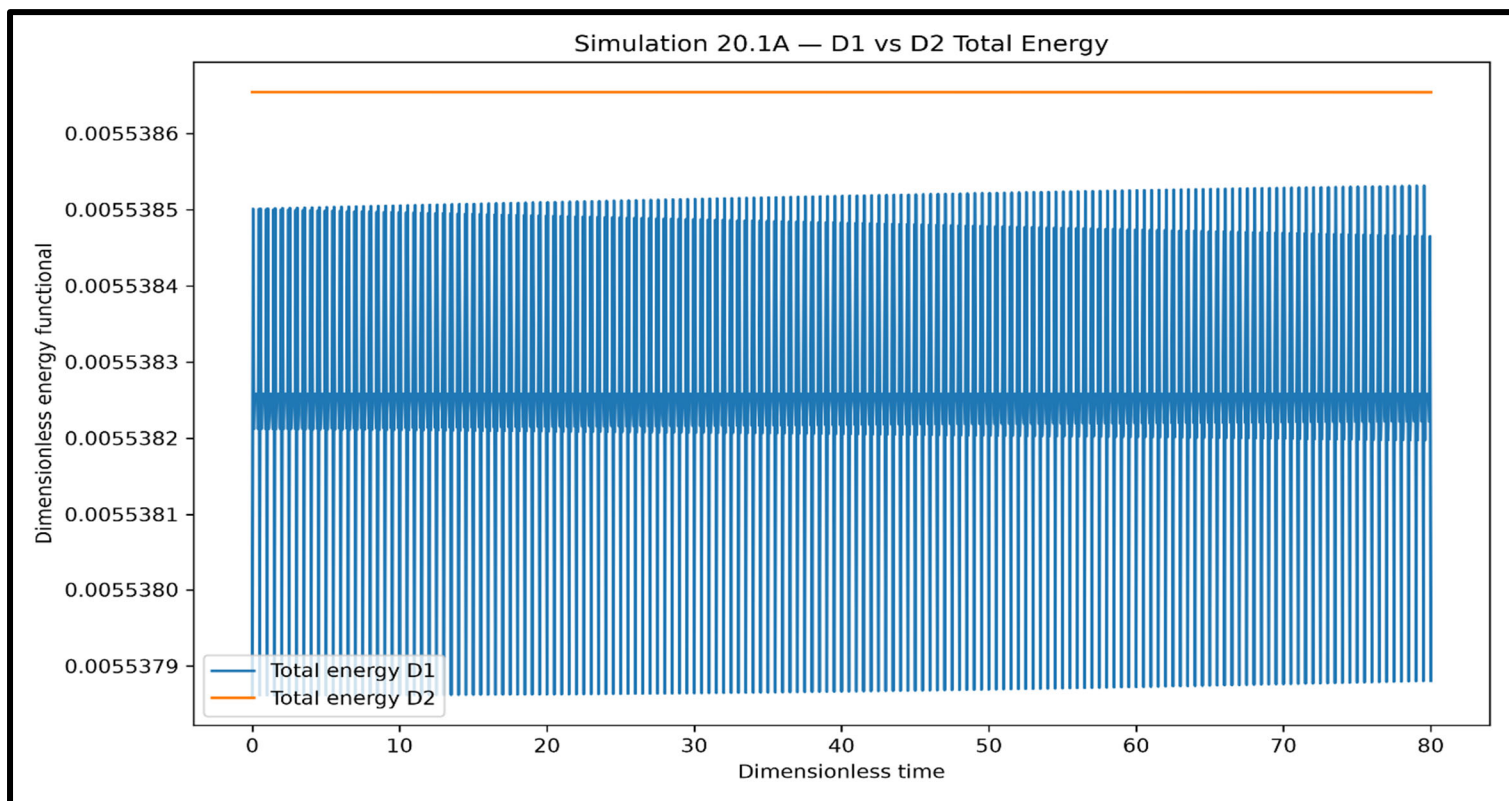


FIGURE 003----Simulation 20.1A

Figure 002 and Figure 003. Simulation 20.1A — Discrete Energy Diagnostic Audit. Two discrete energy representations were evaluated against the unchanged Simulation 20.1 wave equation and numerical integrator. Both diagnostics improved under grid refinement, while the D2 formulation reduced the maximum relative energy error from 1.163960×10^{-8} at $N = 512$ to 1.138828×10^{-11} at $N = 2048$. The audit changed the diagnostic treatment only; no governing physics or integration method was modified.

Simulation 20.2 — Linear Restoring-Term Audit

With Simulation 20.1A frozen, the computational foundation now contained two verified elements: a stable dimensionless wave evolution and a discrete energy diagnostic capable of tracking that evolution with controlled grid-refinement behavior.

The next step was to begin adding physical structure to the otherwise free wave system.

Simulation 20.2 introduced the **first additional term**:

$$-\mu_{\chi}\chi$$

producing the governing equation

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi.$$

The simulation record identifies the new component explicitly as:

Linear restoring term ONLY.

This was intentional. The solver was not advanced directly to the complete nonlinear, damped, forced system. Each physical component would be introduced separately and required to survive numerical audit before another was added.

Objective

The purpose of Simulation 20.2 was to determine whether the validated wave solver could support a linear restoring response without compromising the numerical foundation established by Simulations 20.1 and 20.1A.

The transition was therefore

$$\chi_{tt} = c_{\chi}^2 \chi_{xx}$$

to

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi.$$

The new term causes the field to experience a restoring response proportional to its displacement.

For the simulation,

$$\mu_\chi = 0.35.$$

The associated quantity

$$\sqrt{\mu_\chi}$$

was

$$\sqrt{\mu_\chi} = 0.5916079783.$$

At this stage, these quantities remained part of the **dimensionless numerical model**. No physical IAW reference scale had been assigned.

Addition to the Energy Functional

Introducing a restoring term into the governing equation required a corresponding contribution to the conservative energy.

The new contribution was

$$E_\mu = \frac{1}{2} \mu_\chi \int \chi^2 dx.$$

The energy functional therefore expanded from the wave-only form to

$$E = E_K + E_G + E_\mu$$

where

$$E_K = \frac{1}{2} \int \chi_t^2 dx$$

is the kinetic contribution,

$$E_G = \frac{1}{2} c_\chi^2 \int (\partial_x \chi)^2 dx$$

is the gradient contribution, and

$$E_\mu = \frac{1}{2} \mu_\chi \int \chi^2 dx$$

is the newly introduced restoring contribution.

The initial energy restored calculated by the simulation was

$$E_{\mu,0} = 9.693106997140 \times 10^{-6}.$$

This provided another important rule for the development sequence:

New PDE Term → Corresponding Energy Contribution → Numerical Audit.

The field equation and its energy accounting were being developed together.

Numerical Configuration

Simulation 20.2 inherited its numerical architecture from the frozen 20.1A baseline.

The principal calculation used

$$\begin{aligned} \mathbf{N} &= \mathbf{1024}, \\ \mathbf{L} &= \mathbf{1}, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4}, \end{aligned}$$

and

$$T_{\text{final}} = 80.$$

The propagation coefficient remained

$$c_{\chi} = 1,$$

and the Courant number remained

$$\boxed{\text{CFL} = 0.512}.$$

The Gaussian initial condition also remained unchanged:

$$A_0 = 0.025,$$

$$x_0 = 0.5,$$

$$\sigma = 0.05,$$

With

$$\chi_t(x, 0) = 0.$$

This continuity was important. If several numerical and physical variables had been changed simultaneously, determining which change produced a particular response would have become increasingly difficult.

Instead:

$$\boxed{\text{Frozen Baseline} + \text{One New Component} = \text{Next Audit}.}$$

Energy Evolution

The initial total energies calculated using the two existing diagnostics were

$$E_{D1}(0) = 5.547555060579 \times 10^{-3}$$

and

$$E_{D2}(0) = 5.548347283267 \times 10^{-3}.$$

The restoring-energy component was not static.

At initialization,

$$E_{\mu} = 9.693106997140 \times 10^{-6},$$

and the subsequent simulation showed this energy changing as the field evolved. That behavior was expected from a dynamical restoring system. Energy could move among the kinetic, gradient, and restoring contributions while the conservative total remained controlled. Schematically,

$$\boxed{E_K \leftrightarrow E_G \leftrightarrow E_{\mu}}$$

while

$$E_{\text{total}}$$

should remain conserved to the accuracy of the numerical method. This distinction became increasingly important as additional energy channels were introduced later in the Simulation 20 sequence.

D1 and D2 Diagnostics Remained Active

The diagnostic lesson learned in Simulation 20.1A was carried directly into 20.2. The restoring-term simulation continued to evaluate the solution using both D1 and D2 rather than abandoning the comparison after 20.1A.

For the principal run, D1 produced

$$E_{D1}(80) = 5.547716721103 \times 10^{-3}$$

with a final relative difference of

$$2.914085970519 \times 10^{-5}$$

and a maximum relative difference of

$$1.266547457435 \times 10^{-4}.$$

D2 produced

$$E_{D2}(80) = 5.548347121655 \times 10^{-3},$$

with a final and maximum relative difference of

$$2.912791658682 \times 10^{-8}.$$

The same pattern observed during 20.1A therefore persisted after the restoring term was introduced: the discrete diagnostic treatment had a significant effect on the apparent conservation error.

Grid-Refinement Test

The new restoring-term architecture was then tested at

$$N = 512, 1024, 2048.$$

The corresponding numerical configurations maintained

$$\text{CFL} = 0.512$$

through proportional reduction of the timestep:

N	Δx	Δt	CFL
512	1.953125×10^{-3}	1.0×10^{-3}	0.512
1024	9.765625×10^{-4}	5.0×10^{-4}	0.512
2048	4.882812×10^{-4}	2.5×10^{-4}	0.512

For D1, the maximum relative energy error decreased from

$$2.856273 \times 10^{-4}$$

to

$$1.153950 \times 10^{-4}$$

and finally to

$$2.883963 \times 10^{-5}.$$

Thus,

$$2.856273 \times 10^{-4} \rightarrow 1.153950 \times 10^{-4} \rightarrow 2.883963 \times 10^{-5}.$$

D2 produced:

$$1.163090 \times 10^{-8}$$

at $N = 512$,

$$3.640989 \times 10^{-10}$$

at $N = 1024$,

and

$$1.138511 \times 10^{-11}$$

at $N = 2048$.

Therefore,

$$1.163090 \times 10^{-8} \rightarrow 3.640989 \times 10^{-10} \rightarrow 1.138511 \times 10^{-11}.$$

The error again decreased systematically with increasing resolution.

What Simulation 20.2 Established

Simulation 20.2 demonstrated that the previously validated wave architecture could accept the first additional physical term without destabilizing the calculation. The development sequence had now progressed from

Wave Propagation

to

Wave Propagation + Linear Restoring Response.

The field remained finite. The CFL conditions remained satisfied. The new restoring-energy channel evolved dynamically. The total-energy diagnostics retained controlled behavior. Grid refinement reduced the measured numerical error. Most importantly, the result was obtained without introducing the next physical mechanism prematurely.

No Physical Calibration Was Claimed

Simulation 20.2 did **not** establish that

$$\mu_{\chi} = 0.35$$

was a physical constant of the IAW. The simulation remained dimensionless. No dimensional reference length, time, or field amplitude was assigned. Likewise, no direct identification between the solver coefficient μ_{χ} and the manuscript parameter μ was assumed at this stage. That separation was deliberate.

The immediate problem was:

Can the numerical architecture correctly support this term?

not:

What physical value must this coefficient possess inside a black hole?

Physical calibration would require additional theoretical constraints and was not allowed to contaminate the numerical qualification process.

Simulation 20.2 Disposition

Simulation 20.2 successfully introduced the linear restoring term while retaining the numerical foundation inherited from frozen Simulation 20.1A.

The resulting progression was therefore:

$$\boxed{20.1 \rightarrow 20.1A \rightarrow 20.2}$$

or, in terms of mathematical architecture,

$$\boxed{\chi_{tt} = c_{\chi}^2 \chi_{xx}}$$

became

$$\boxed{\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi.}$$

Simulation 20.2 was subsequently retained as a frozen component of the solver architecture; the later scale-invariance audit explicitly identifies the **frozen 20.2 term** as

$$-\mu_{\chi} \chi.$$

The next development question was now clear:

Could the solver support nonlinear self-interaction while preserving the numerical behavior already established?

That question became **Simulation 20.3 — Cubic Nonlinear Self-Interaction Audit.**

FIGURE 004 — Simulation 20.2.

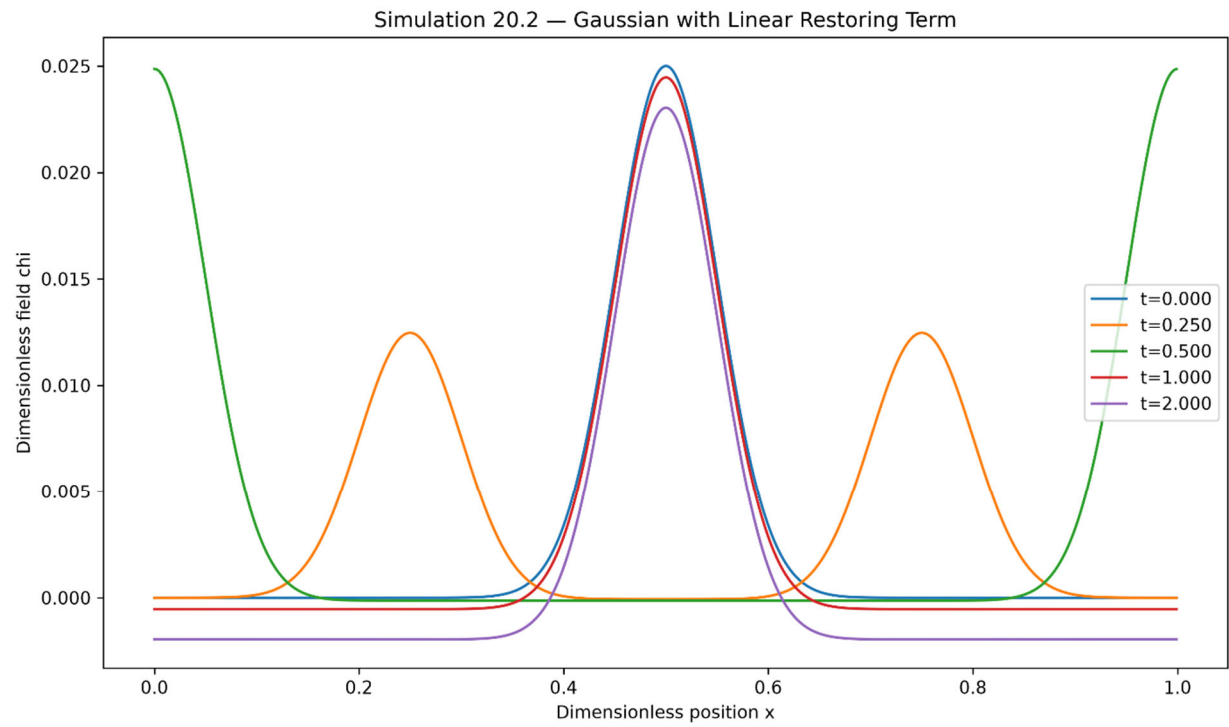


FIGURE 005 — Simulation 20.2

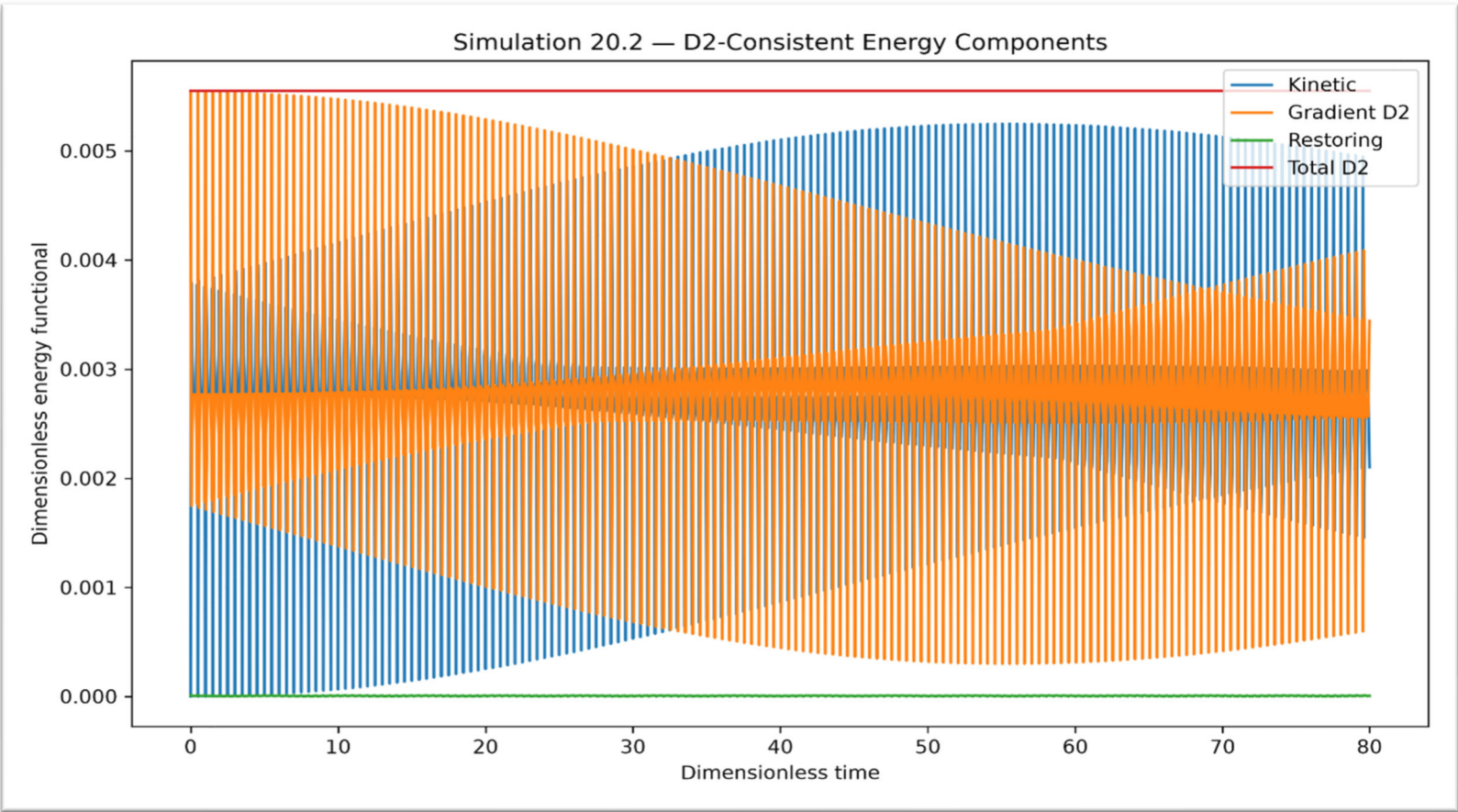
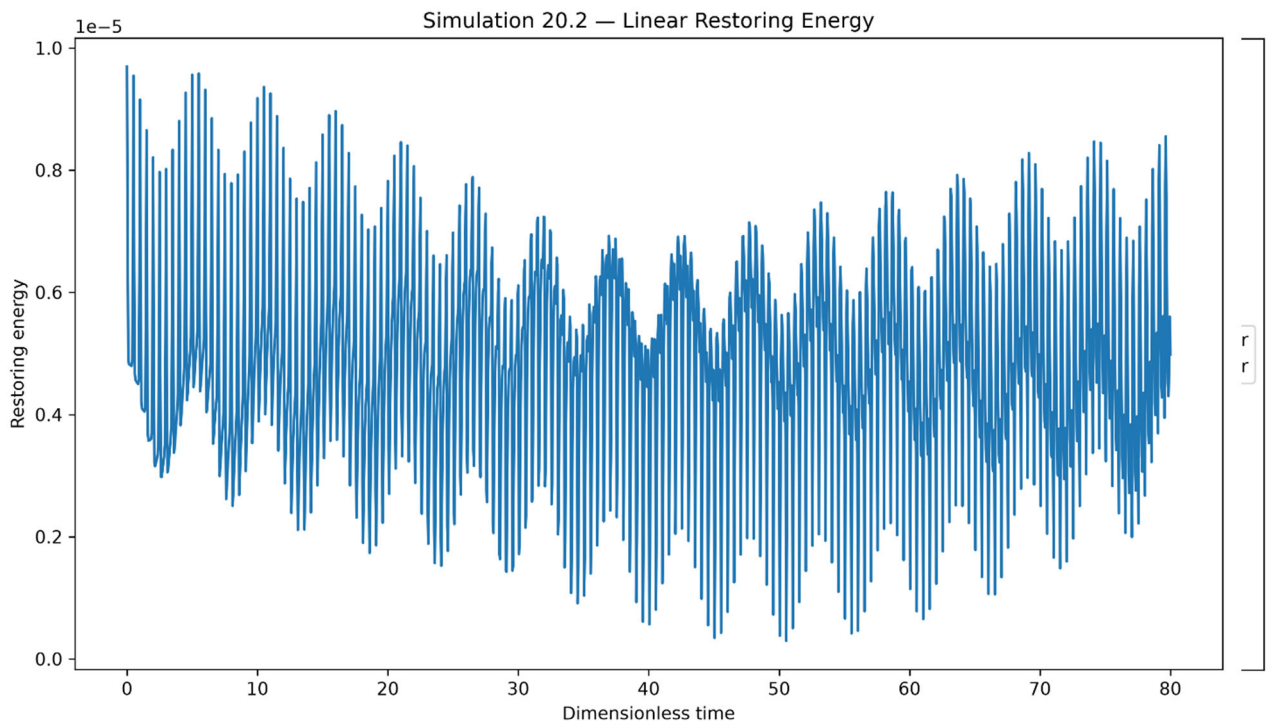


FIGURE 006 — Simulation 20.2

FIGURE 007 — Simulation 20.2

Figures 004-007. Simulation 20.2 — Linear Restoring-Term Audit. The first additional field component, $-\mu_\chi\chi$, was introduced into the frozen wave baseline together with its corresponding restoring-energy contribution. The solution remained finite and stable, while refinement from $N = 512$ to $N = 2048$ systematically reduced the measured energy error. The simulation remained dimensionless and introduced no physical IAW scale assignment.



Simulation 20.3 — Cubic Nonlinear Self-Interaction Audit

With the linear restoring response successfully incorporated and Simulation 20.2 frozen, the solver was ready for its first genuine nonlinear component.

Simulation 20.3 introduced the cubic self-interaction term

$$-\lambda_{\chi}\chi^3$$

into the established field equation. The governing system became

$$\chi_{tt} = c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi - \lambda_{\chi}\chi^3.$$

The execution record identifies the new component specifically as “**Cubic self-interaction ONLY.**” This represented an important transition. Simulations 20.1 through 20.2 had established a linear computational system. Simulation 20.3 tested whether that architecture remained numerically controlled once the restoring response became dependent on the field amplitude itself.

Objective

The purpose of Simulation 20.3 was to answer:

Can the frozen linear solver support cubic nonlinear self-interaction while maintaining finite field evolution and controlled energy behavior?

The progression was deliberately restricted to one new term:

$$\underbrace{c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi}_{\text{Frozen 20.2 architecture}} \rightarrow \underbrace{c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi - \lambda_{\chi}\chi^3}_{\text{Simulation 20.3}}$$

with

$$\lambda_{\chi} = 20.$$

The existing propagation and linear restoring components were retained unchanged.

Why the Cubic Term Matters

The linear restoring term produces a response proportional to field amplitude:

$$F_{\text{linear}} \propto -\mu_{\chi}\chi.$$

The cubic term changes this relationship:

$$F_{\text{nonlinear}} \propto -\lambda_{\chi}\chi^3.$$

Consequently, the restoring behavior is no longer strictly proportional to χ . At small amplitudes, the cubic contribution can remain comparatively small. As the magnitude of the field increases, however,

$$|\chi|^3$$

grows more rapidly than

$$|\chi|.$$

The solver therefore had to evolve an amplitude-dependent restoring system rather than a purely linear wave. This was the first point in the Simulation 20 development sequence at which the field equation became explicitly nonlinear.

Corresponding Nonlinear Energy

As in Simulation 20.2, introducing a new conservative PDE term required introducing its corresponding contribution to the energy functional.

For

$$-\lambda_{\chi}\chi^3,$$

the quartic energy contribution is

$$E_{\lambda} = \frac{1}{4}\lambda_{\chi} \int \chi^4 dx.$$

The conservative energy architecture therefore expanded to

$$E = E_K + E_G + E_\mu + E_\lambda.$$

The initial nonlinear energy calculated by Simulation 20.3 was

$$E_{\lambda,0} = 1.223939587222 \times 10^{-7}.$$

For comparison, the initial restoring energy remained

$$E_{\mu,0} = 9.693106997140 \times 10^{-6}.$$

Thus, the nonlinear contribution began much smaller than the linear restoring contribution for the selected initial field amplitude. It was nevertheless dynamically active.

Numerical Configuration

Simulation 20.3 explicitly identifies **Simulation 20.2 LOCKED** as its baseline.

The principal numerical configuration remained

$$\begin{aligned} N &= 1024, L = 1, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4}, \end{aligned}$$

and

$$T_{\text{final}} = 80.$$

The coefficients were

$$\begin{aligned} c_\chi &= 1, \\ \mu_\chi &= 0.35, \end{aligned}$$

and the newly introduced

$$\lambda_\chi = 20.$$

The Courant number remained

$$\boxed{\text{CFL} = 0.512}.$$

The same Gaussian initial disturbance was retained:

$$A_0 = 0.025, x_0 = 0.5, \sigma = 0.05,$$

with zero initial velocity. Again, no physical reference scales were assigned.

The Conservative Architecture Had Now Become Nonlinear

The progression through the first three principal stages can now be written directly.

Simulation 20.1

$$\boxed{\chi_{tt} = c_\chi^2 \chi_{xx}}$$

Simulation 20.2

$$\boxed{\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi}$$

Simulation 20.3

$$\boxed{\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3}$$

The corresponding energy architecture evolved in parallel:

$$E_K + E_G$$

became

$$E_K + E_G + E_\mu$$

and now

$$\boxed{E_K + E_G + E_\mu + E_\lambda.}$$

This staged construction provided direct traceability between every term in the PDE and every contribution included in the conservative energy accounting.

Early Evolution of Nonlinear Energy

The recorded simulation output shows that E_λ changed continuously as the field evolved.

Initially,

$$E_\lambda = 1.223939587222 \times 10^{-7}.$$

At $t = 5$,

$$E_\lambda = 3.770 \times 10^{-8},$$

and at $t = 10$,

$$E_\lambda = 1.117 \times 10^{-7}.$$

At $t = 15$,

$$E_\lambda = 3.443 \times 10^{-8},$$

while the linear restoring contribution also evolved over the same interval. This oscillatory exchange was significant. The nonlinear energy term was not merely present algebraically in the code. It participated dynamically in the evolving energy distribution. The conservative system now contained multiple interacting storage mechanisms:

$$\boxed{E_K \leftrightarrow E_G \leftrightarrow E_\mu \leftrightarrow E_\lambda.}$$

What Simulation 20.3 Was Testing

The central issue was not simply whether Python could numerically evaluate χ^3 .

The nonlinear term changes the mathematical character of evolution.

The solver now had to demonstrate that:

- the field remained finite;
- the nonlinear term produced a calculable response;

- the corresponding quartic energy was represented;
- the previously established terms remained intact;
- numerical energy behavior remained controlled; and
- increasing resolution continued to improve the numerical solution rather than expose instability.

This distinction matters because nonlinear terms can amplify numerical problems that remain difficult to detect in a purely linear calculation. Simulation 20.3 therefore represented a substantially stronger test of the evolving solver architecture than the preceding linear cases.

No Physical Calibration Was Introduced

As with μ_χ , the value

$$\lambda_\chi = 20$$

was not declared to be a measured physical constant of the Interior Accumulation Web. The solver remained explicitly dimensionless, and the execution record states:

Reference Scales: Not assigned.

The purpose remained numerical construction. At this stage the question was:

Can the mathematical architecture support nonlinear self-interaction?

The physical mapping of dimensionless coefficients to a dimensional IAW constitutive state remained a separate problem. That separation prevented an assumed physical scale from being inserted merely to make the numerical system operational.

What Simulation 20.3 Added to the ICSO Development

The solver had now crossed an important threshold. It was no longer simply a wave solver with a linear restoring force. It contained a nonlinear constitutive response. The development chain had become

20.1 → 20.1A → 20.2 → 20.3.

In physical architecture:

Propagation → Validated Energy Diagnostic → Linear Restoring Response → Nonlinear Self-Interaction.

The progression remained controlled because each new component was introduced only after the preceding architecture had been frozen.

Simulation 20.3 Disposition

Simulation 20.3 established the cubic nonlinear term as part of the retained solver architecture. This is independently reflected later in the Simulation 20.7A theory-control record, which identifies

$$-\lambda_{\chi}\chi^3$$

as the **locked Simulation 20.3 term**, alongside the other previously frozen components. The solver architecture had therefore reached

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi - \lambda_{\chi} \chi^3.$$

and Simulation 20.3 became part of the frozen computational foundation. The next challenge was fundamentally different. The simulations to this point were conservative. Energy could move among kinetic, gradient, linear-restoring, and nonlinear storage channels, but no explicit mechanism had yet been introduced to remove energy from the evolving field.

The next stage would introduce **dissipation**.

And because that stage required a corrective revision, the historical sequence should preserve both versions:

$$20.3 \rightarrow 20.4 \rightarrow 20.4A$$

rather than jumping directly to the final corrected result.

That begins with **Simulation 20.4 — Linear Damping Audit**.

FIGURE 008— Simulation 20.3

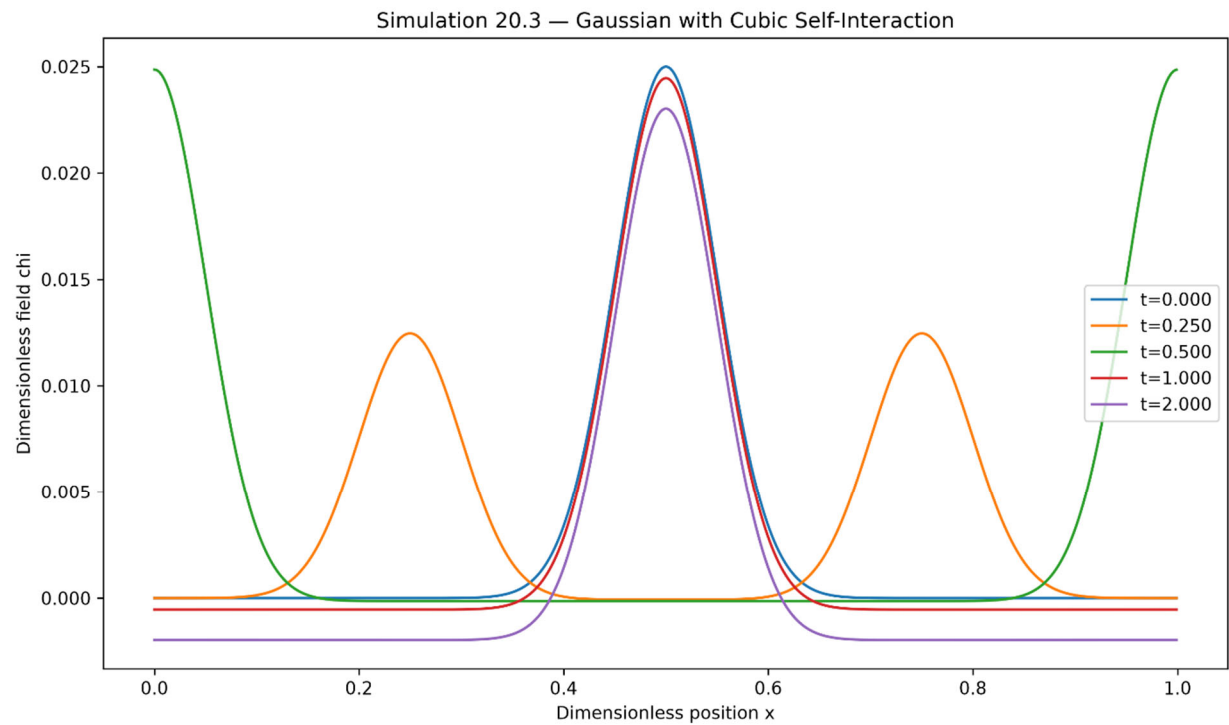


FIGURE 009— Simulation 20.3

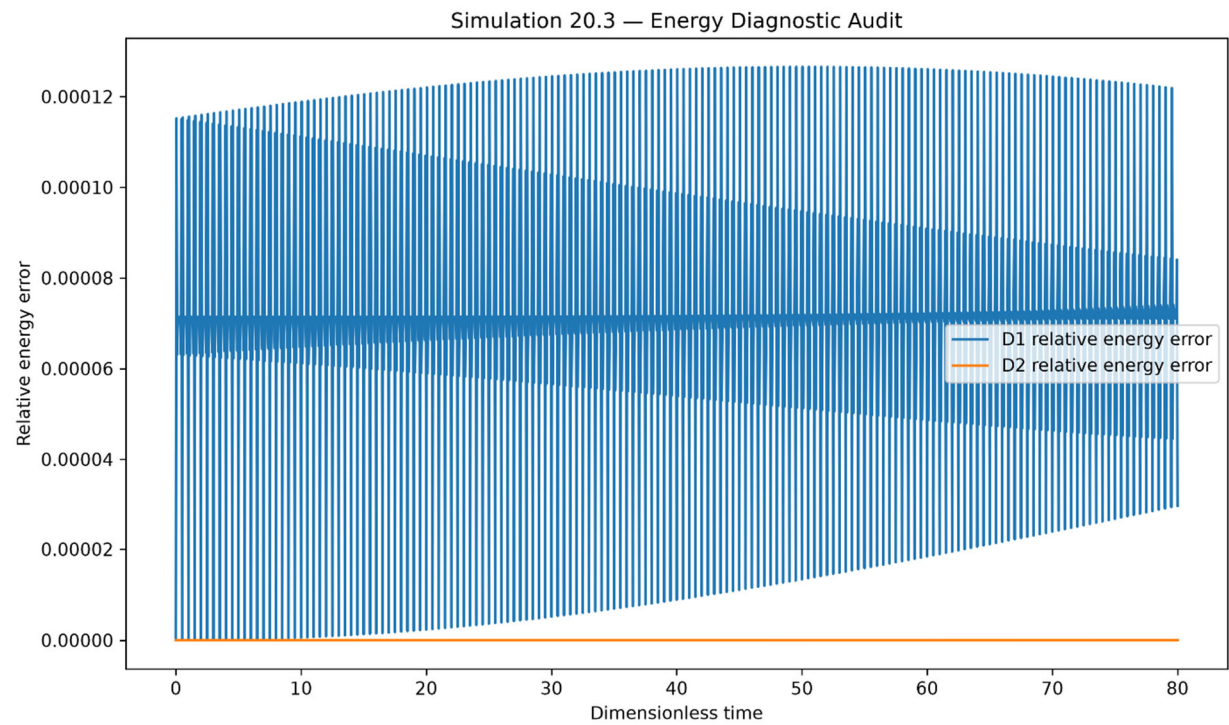
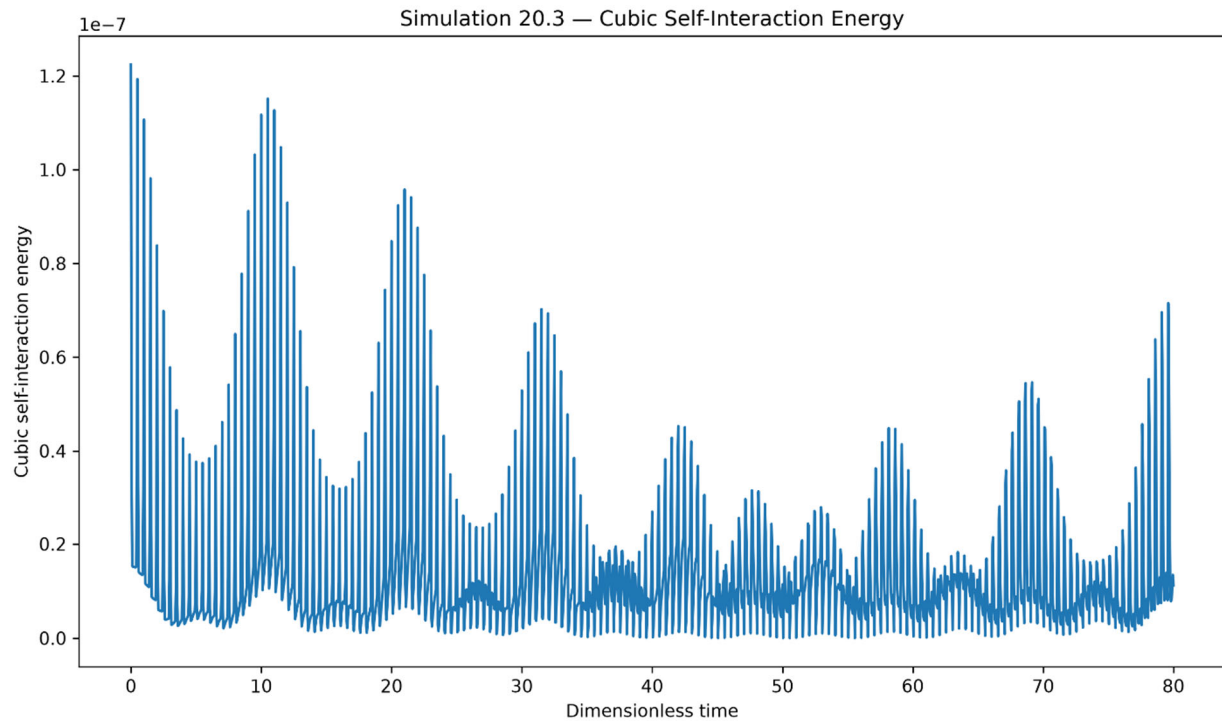


FIGURE 010— Simulation 20.3



Figures 008-010. Simulation 20.3 — Cubic Nonlinear Self-Interaction Audit. The cubic term $-\lambda_\chi \chi^3$ was introduced into the frozen Simulation 20.2 architecture together with its corresponding quartic energy contribution $E_\lambda = (1/4)\lambda_\chi \int \chi^4 dx$. The resulting solver represented the first nonlinear stage of the Simulation 20 development program while retaining the previously established propagation and linear restoring components.

Simulation 20.4 — Linear Damping Audit

Simulation 20.3 established the first nonlinear conservative solver architecture. The system could propagate, support a linear restoring response, and support cubic nonlinear self-interaction while retaining explicit accounting of the corresponding energy-storage mechanisms. Simulation 20.4 introduced a fundamentally different component:

$$\boxed{-\gamma_{\chi}\chi_t}$$

the first **dissipative term** in the Simulation 20 development sequence. The governing equation became

$$\boxed{\chi_{tt} = c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi - \lambda_{\chi}\chi^3 - \gamma_{\chi}\chi_t.}$$

The Simulation 20.4 record identifies Simulation 20.3 as the frozen baseline and identifies **linear damping only** as the new component. This simulation produced an important development event: **the field behaved correctly, but the original validation metric did not**. That distinction forced the creation of Simulation 20.4A.

Objective

The purpose of Simulation 20.4 was to determine whether linear dissipation could be added to the existing nonlinear field architecture. The new coefficient was

$$\boxed{\gamma_{\chi} = 0.02.}$$

The expected energy law was no longer

$$\frac{dE}{dt} = 0.$$

Instead,

$$\boxed{\frac{dE}{dt} = -\gamma_{\chi} \int \chi_t^2 dx.}$$

The Simulation 20.4 theory-control record explicitly states this expected energy law and confirms that no nonlinear damping, forcing, or constitutive analyzer was added. This changed the meaning of energy loss completely. For the first time,

decreasing field energy $\neq$ numerical error.

Energy was now *supposed* to leave the resolved field.

Numerical Configuration

The established baseline configuration was retained:

$$\begin{aligned} N &= 1024, L = 1, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4}, \\ T_{\text{final}} &= 80, \end{aligned}$$

with

$$\text{CFL} = 0.512.$$

The coefficients were

$$\begin{aligned} c_\chi &= 1, \mu_\chi = 0.35, \\ \lambda_\chi &= 20, \boxed{\gamma_\chi = 0.02}. \end{aligned}$$

The same Gaussian initial condition remained in place:

$$A_0 = 0.025, x_0 = 0.5, \sigma = 0.05,$$

with zero initial velocity.

Again:

Frozen Baseline + One New Physical Component.

Energy Was Expected to Decrease

The initial D2 total energy was

$$E_{D2}(0) = 5.548469677226 \times 10^{-3}.$$

At

$$t = 80,$$

it had fallen to

$$E_{D2}(80) = 1.120524202676 \times 10^{-3}.$$

The raw relative change was therefore approximately

$$\boxed{0.7980480623}$$

or about **79.8%**.

Under the earlier conservative tests, such a number would have represented catastrophic failure. But Simulation 20.4 was not conservative. The governing equation explicitly contained

$$-\gamma_{\chi}\chi_t.$$

Therefore, the declining energy was a physical consequence of the equation being solved. This was precisely where the original audit logic broke down.

The Initial Audit Was Asking the Wrong Question

The original Simulation 20.4 grid-convergence test continued to treat

$$\frac{|E(t) - E(0)|}{E(0)}$$

as though it represented numerical conservation error. The resulting grid values were approximately

$$1.978417 \times 10^{-2},$$

$$1.978280 \times 10^{-2},$$

and

$$1.978246 \times 10^{-2}$$

for $N = 512, 1024, 2048$, respectively. But the quantity being measured contained **real damping loss**. Refining the grid could not drive that loss toward zero because the governing physics explicitly required the field to dissipate energy. This exposed a critical validation mistake:

Physical Energy Loss $\neq$ Numerical Conservation Error.

The solver did not need to be changed. The audit did.

What Simulation 20.4 Actually Revealed

The raw calculation itself behaved coherently. The field remained finite. The CFL test passed. The restoring energy decreased from

$$9.693106997140 \times 10^{-6}$$

to

$$1.004819904456 \times 10^{-6},$$

while the cubic energy decreased from

$$1.223939587222 \times 10^{-7}$$

to

$$4.656872819713 \times 10^{-10}.$$

The problem was not an exploding solution or unstable field. The problem was that the existing conservation test had been inherited from a conservative system and applied to a dissipative one. The correct quantity to test was no longer simply

$$E(t).$$

The energy removed through damping had to be accounted for.

Correct Energy Accounting

From

$$\frac{dE}{dt} = -\gamma_\chi \int \chi_t^2 dx,$$

define cumulative linear damping loss

$$L_\gamma(t) = \int_0^t \gamma_\chi \int \chi_t^2 dx dt'.$$

Then the appropriate compensated quantity becomes

$$E_{\text{comp}}(t) = E(t) + L_\gamma(t).$$

For a correctly implemented dissipative solver,

$$E(t)$$

should decrease, while

$$E(t) + L_\gamma(t) \approx E(0)$$

should remain numerically controlled. This changed the validation question from

Did the field energy remain constant?

to

Does the calculated reduction in field energy agree with the energy removed by the damping law?

That is a much stronger and physically correct test.

Simulation 20.4 Disposition

Simulation 20.4 therefore produced a valuable intermediate result. The linear damping physics itself was not rejected. The test exposed a flaw in diagnostic **architecture**. The appropriate development record is therefore

20.4 → Diagnostic Problem Identified → 20.4A.

The physical equation was retained:

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi - \lambda_{\chi} \chi^3 - \gamma_{\chi} \chi_t.$$

But the validation method required correction before the damping term could be frozen.

Accordingly:

SIMULATION 20.4 — REVISED

rather than treating the first damping implementation as the final validated result.

Simulation 20.4A — Compensated Linear-Damping Energy Audit

Simulation 20.4A was created to correct the diagnostic problem exposed by 20.4. Critically, **20.4A did not alter the physics**. The governing equation remained

$$\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3 - \gamma_\chi \chi_t.$$

The 20.4A theory-control record states explicitly: “20.4 physics is unchanged.” and:

“20.4A changes diagnostics only: damped-grid convergence now tests compensated energy balance.” This was another example of the principle first encountered in Simulation 20.1A:

Do not modify the physics until you know the physics is what failed.

Compensated Energy Balance

Simulation 20.4A calculated the cumulative damping loss directly. At the end of the $T = 80$ calculation,

$$L_\gamma = 4.427945415146 \times 10^{-3}.$$

The final field energy was

$$E_{D2}(80) = 1.120524202676 \times 10^{-3}.$$

Adding the calculated damping loss produced

$$E_{\text{comp}}(80) = 5.548469617821 \times 10^{-3}.$$

The initial D2 energy had been

$$E_{D2}(0) = 5.548469677226 \times 10^{-3}.$$

The resulting final damping-balance error was only

$$1.070647650969 \times 10^{-8},$$

with a maximum balance error over the calculation of

$$1.514491904279 \times 10^{-8}.$$

The apparent 79.8% “energy error” had therefore been correctly identified. It was not a 79.8% numerical failure. It was primarily the physical energy removed by the damping term.

Grid Convergence of the Correct Quantity

Simulation 20.4A then repeated the resolution study using the **compensated damping-law balance** rather than raw energy loss.

At the common comparison time $T = 10$:

N	Raw Physical Loss	Max Compensated Balance Error
512	1.811724×10^{-1}	1.092447×10^{-7}
1024	1.811608×10^{-1}	8.593183×10^{-9}
2048	1.811579×10^{-1}	4.050625×10^{-9}

The physical damping loss remained approximately the same as the grid was refined.

That was expected. The numerical balance error, however, decreased substantially:

$$1.092447 \times 10^{-7} \rightarrow 8.593183 \times 10^{-9} \rightarrow 4.050625 \times 10^{-9}.$$

This was the behavior the audit needed to test.

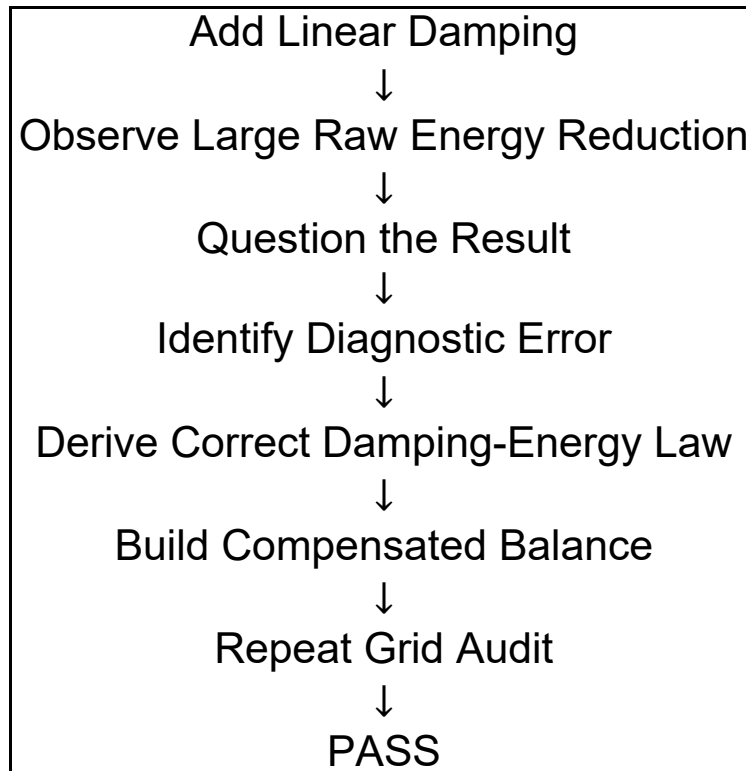
Why 20.4 and 20.4A Belong in the Manuscript

This pair is an excellent example of why we decided not to omit the intermediate simulations. If the manuscript showed only 20.4A, the development history would appear straightforward:

Add damping → PASS.

That is not what happened.

The actual sequence was more informative:



Nothing was hidden and no tolerance was loosened to manufacture a passing result. The quantity being tested was corrected because the previous quantity was physically inappropriate for a dissipative system.

Simulation 20.4A Disposition

The corrected audit demonstrated that the linear damping implementation obeyed its expected energy-loss law to high numerical accuracy. The later Simulation 20.7 validation record confirms that the retained solver architecture treated

$$-\gamma_{\chi}\chi_t$$

as the **frozen Simulation 20.4A term**. The development chain therefore became

20.1 → 20.1A → 20.2 → 20.3 → 20.4 → 20.4A.

with

SIMULATION 20.4A — LOCKED

and the solver equation now stood at

$$\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3 - \gamma_\chi \chi_t.$$

The next step would make dissipation itself nonlinear. That became **Simulation 20.5 — Nonlinear Damping Audit.**

20.4A

Grid convergence:

N=2048 dx=4.882812e-04 dt=2.500000e-04 CFL=0.512000 D1 max
dE/E=1.978246e-02 D2 max dE/E=1.978246e-02

20.4

Grid convergence — compensated damping-law balance (common T=10):

N=2048 dx=4.882812e-04 dt=2.500000e-04 CFL=0.512000 raw physical
loss=1.811579e-01 max balance err=4.050625e-09

Simulations 20.4 and 20.4A — Linear Damping and Corrected Energy Accounting.

Introduction of the linear damping term produced a large reduction in raw field energy, initially making the inherited conservative energy metric unsuitable as a numerical-error measure. Simulation 20.4A retained the governing physics unchanged and incorporated cumulative damping loss into the diagnostic. The compensated energy balance reduced the maximum residual to 1.51449×10^{-8} , demonstrating that the observed energy reduction represented the prescribed dissipative physics rather than numerical energy loss.

Simulation 20.5 — Nonlinear Damping Audit

With Simulation 20.4A frozen, the solver now contained propagation, linear restoring response, cubic nonlinear self-interaction, and a validated linear dissipation mechanism. Simulation 20.5 introduced the next physical component:

$$\boxed{-\eta_{\chi}\chi^2\chi_t}$$

the first **nonlinear damping term** in the Simulation 20 development sequence. The governing equation became

$$\boxed{\chi_{tt} = c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi - \lambda_{\chi}\chi^3 - \gamma_{\chi}\chi_t - \eta_{\chi}\chi^2\chi_t.}$$

The Simulation 20.5 execution record identifies **Simulation 20.4A LOCKED** as its baseline and **nonlinear damping only** as the new component. This extended the lessons learned during 20.4A. Energy loss could no longer be treated merely as a change in total field energy. Each dissipative mechanism had to be calculated and accounted for independently.

Objective

Simulation 20.5 asked, **Can an amplitude-dependent nonlinear damping mechanism be added to the frozen linearly damped solver while preserving finite evolution and satisfying the corresponding dissipative energy law?**

The frozen linear damping term remained

$$-\gamma_{\chi}\chi_t,$$

while the new term was

$$-\eta_{\chi}\chi^2\chi_t.$$

For the simulation,

$$\boxed{\eta_{\chi} = 80}.$$

The distinction between the two mechanisms is important.

Linear damping depends on velocity:

$$F_{\text{lin,damp}} \propto -\gamma_{\chi} \chi_t.$$

Nonlinear damping depends on both velocity and field amplitude:

$$F_{\text{nl,damp}} \propto -\eta_{\chi} \chi^2 \chi_t.$$

Consequently, the nonlinear dissipative response becomes stronger in regions where $|\chi|$ is larger.

The Energy Law Expanded Again

Simulation 20.4A had established

$$\frac{dE}{dt} = -\gamma_{\chi} \int \chi_t^2 dx.$$

With nonlinear damping added, Simulation 20.5 used

$$\boxed{\frac{dE}{dt} = - \int (\gamma_{\chi} + \eta_{\chi} \chi^2) \chi_t^2 dx.}$$

The execution record explicitly states this expected energy law. The two cumulative loss channels could therefore be separated.

Linear damping loss

$$\boxed{L_{\text{lin}}(t) = \int_0^t \gamma_{\chi} \int \chi_t^2 dx dt'}$$

Nonlinear damping loss

$$\boxed{L_{\text{nl}}(t) = \int_0^t \eta_{\chi} \int \chi^2 \chi_t^2 dx dt'.$$

The compensated energy relation became

$$E(t) + L_{\text{lin}}(t) + L_{\text{nl}}(t) \approx E(0).$$

The solver was therefore no longer merely required to lose energy. It had to calculate **where that energy went according to the mathematical model.**

Numerical Configuration

The principal calculation retained the established numerical baseline:

$$\begin{aligned} N &= 1024, L = 1, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4}, \\ T_{\text{final}} &= 80, \end{aligned}$$

with

$$\text{CFL} = 0.512.$$

The coefficients were now

$$\begin{aligned} c_\chi &= 1, \\ \mu_\chi &= 0.35, \\ \lambda_\chi &= 20, \\ \gamma_\chi &= 0.02, \end{aligned}$$

and

$$\eta_\chi = 80.$$

The Gaussian initial condition remained

$$A_0 = 0.025, x_0 = 0.5, \sigma = 0.05,$$

with zero initial velocity.

Once again:

$$\text{Locked Architecture} + \text{One New Physical Component.}$$

Nonlinear Damping Was Dynamically Active

The nonlinear damping term was not simply inserted into the equation and assumed to matter. Its energy removal was calculated separately from the linear damping loss.

At

$$t = 5,$$

the reported nonlinear-loss fraction was approximately

$$\boxed{0.1752},$$

or about **17.5% of the accumulated damping loss** at that point.

At

$$t = 10,$$

the nonlinear fraction was approximately

$$0.1537,$$

and at

$$t = 20,$$

approximately

$$0.1406.$$

By

$$t = 30,$$

the reported nonlinear-loss fraction was approximately

$$0.1293.$$

This declining fraction is consistent with the mathematical form of the term.

Because

$$L_{\text{nl}} \propto \chi^2 \chi_t^2,$$

The nonlinear damping contribution is amplitude dependent. As damping reduces the field amplitude, the χ^2 factor reduces the relative strength of that dissipative channel. Thus the solver produced a response inherent to the equation rather than applying a constant additional loss rate.

Compensated Balance Remained Controlled

At $t = 5$, the raw energy loss was already approximately

$$1.138 \times 10^{-1},$$

while the reported compensated balance error was only

$$\boxed{9.548 \times 10^{-10}}.$$

At $t = 10$,

$$\text{raw loss} \approx 2.099 \times 10^{-1},$$

while

$$\text{balance error} \approx 2.698 \times 10^{-9}.$$

At $t = 20$,

$$\text{raw loss} \approx 3.706 \times 10^{-1},$$

while the balance residual remained approximately

$$1.864 \times 10^{-9}.$$

This distinction is crucial:

$$\boxed{\text{Large Physical Energy Loss and Small Numerical Balance Error}}$$

can coexist. Simulation 20.4A had established the correct way to interpret a dissipative system. Simulation 20.5 demonstrated that the same principle could be extended to multiple, mathematically distinct damping channels.

The Solver Could Now Distinguish Loss Mechanisms

Before Simulation 20.5, all prescribed dissipation came from

$$-\gamma_{\chi}\chi_t.$$

After 20.5, the solver separately tracked

$$L_{\text{lin}}$$

and

$$L_{\text{nl}}.$$

This produced a more informative energy architecture:

$$E_K + E_G + E_{\mu} + E_{\lambda}$$

for instantaneous stored field energy, together with

$$L_{\text{lin}} + L_{\text{nl}}$$

for cumulative dissipative loss.

The accounting identity was now

$$E(t) + L_{\text{lin}}(t) + L_{\text{nl}}(t) \approx E(0).$$

This was an important advance toward the later solver architecture because energy could now be categorized by mechanism rather than merely reported as a single declining quantity.

What Simulation 20.5 Established

Simulation 20.5 extended the solver in three important ways.

First, dissipation became **state dependent** through the χ^2 factor.

Second, linear and nonlinear loss mechanisms were tracked independently.

Third, the compensated energy diagnostic established in Simulation 20.4A remained applicable after the additional dissipative channel was introduced.

The solver architecture had therefore progressed to

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi - \lambda_{\chi} \chi^3 - \gamma_{\chi} \chi_t - \eta_{\chi} \chi^2 \chi_t.$$

This contained propagation, linear and nonlinear restoring behavior, and linear and nonlinear dissipation.

Important Theory Boundary

There is an important distinction that should remain visible in the manuscript.

The nonlinear damping term

$$-\eta_{\chi} \chi^2 \chi_t$$

became part of the **frozen computational solver architecture** developed through the Simulation 20 sequence. However, it is **not presently an explicit term in the approved Chapter 2 governing IAW equation**, which currently contains the linear damping contribution

$$\gamma \partial_t \Psi$$

but no corresponding $\eta \Psi^2 \partial_t \Psi$ term. That difference should not be silently erased.

Accordingly, Simulation 20.5 establishes that the numerical architecture can support and validate nonlinear damping. It does **not**, by itself, authorize insertion of that term into the governing manuscript theory. The relationship between the legacy/frozen nonlinear damping test and the final constitutive theory remains a separate theory decision.

This preserves the distinction between

Computational Capability

and

Approved IAW Physics.

Simulation 20.5 Disposition

The subsequent simulation record identifies Simulation 20.5 as the frozen baseline component supplying

$$-\eta_{\chi}\chi^2\chi_t.$$

For example, the later Simulation 20.7 theory-control record explicitly lists the **locked 20.5 term** alongside the previously frozen restoring, cubic, and linear-damping terms. The development chain therefore became

20.1 → 20.1A → 20.2 → 20.3 → 20.4 → 20.4A → 20.5.

with

SIMULATION 20.5 — LOCKED

The solver could now remove energy through two independently calculated dissipative mechanisms. What it could not yet do was **receive energy from an external source**. That became the next test.

Simulation 20.6 — External Forcing and Full Energy-Balance Audit

FIGURE 011 — Simulation 20.5

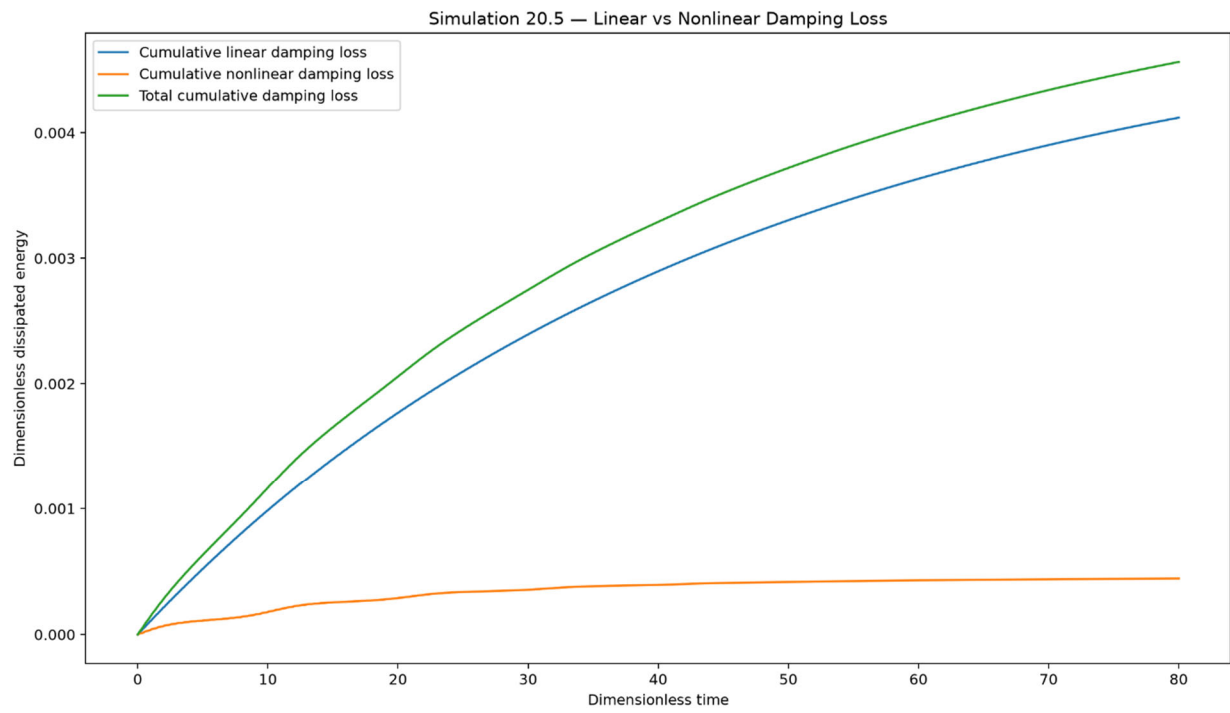


FIGURE 012 — Simulation 20.5

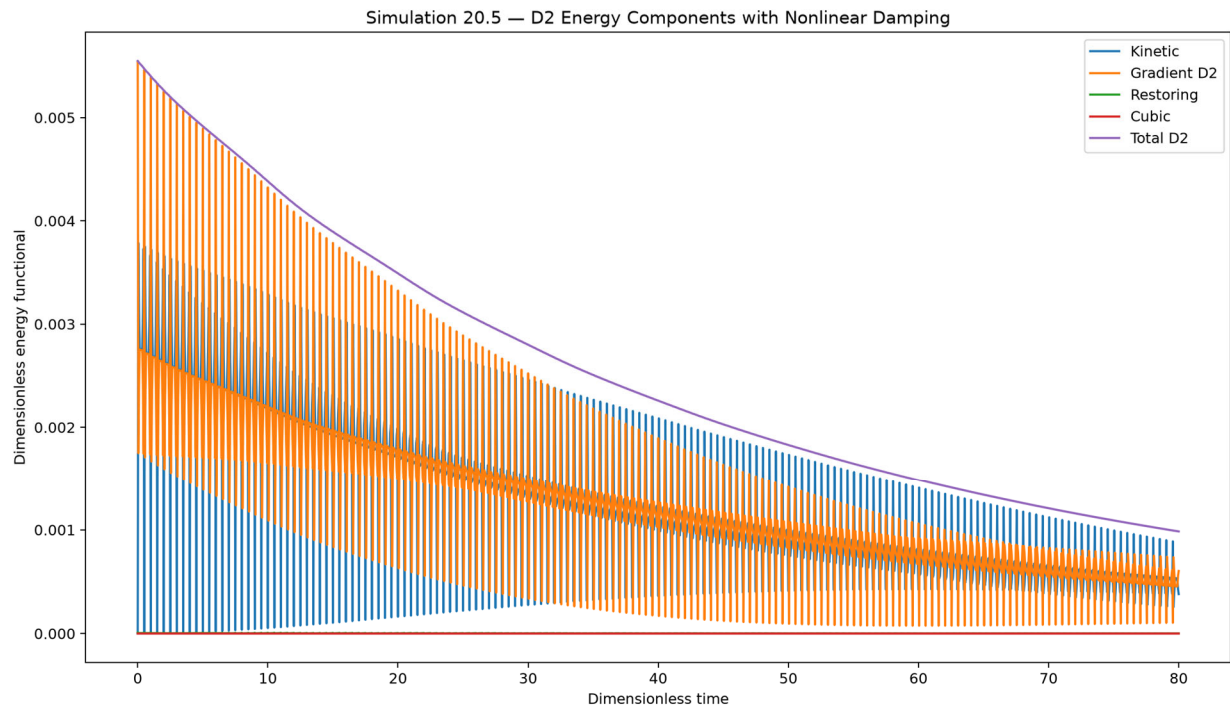


FIGURE 013 — Simulation 20.5

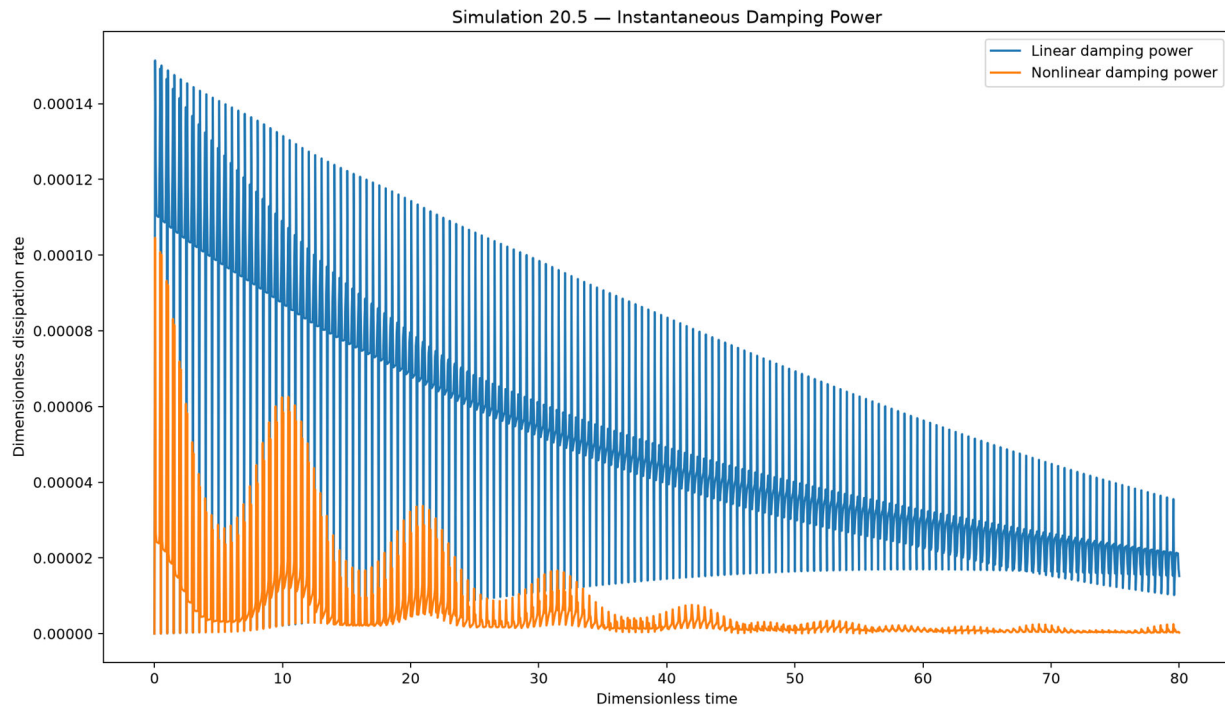


Figure 011-013. Simulation 20.5 — Nonlinear Damping Audit. The nonlinear damping term $-\eta_{\chi}\chi^2\chi_t$ was added to the frozen Simulation 20.4A architecture and tracked independently from linear damping. The nonlinear contribution was strongest while the field amplitude remained comparatively large and decreased in relative importance as the field dissipated. Accounting for both cumulative loss channels preserved the compensated energy balance while allowing substantial physical energy removal from the evolving field.

Simulation 20.6 — External Forcing Audit

Simulation 20.5 completed the dissipative portion of the developing solver. The field could propagate, respond through linear and nonlinear restoring mechanisms, and dissipate energy through independently tracked linear and nonlinear damping channels.

The next test reversed the direction of energy transfer.

For the first time, the solver was required to **accept externally supplied energy** while simultaneously calculating internal storage and dissipation.

Simulation 20.6 introduced one new term:

$$+F_{\chi}\sin(\omega_{\chi}t)$$

producing

$$\chi_{tt} = c_{\chi}^2\chi_{xx} - \mu_{\chi}\chi - \lambda_{\chi}\chi^3 - \gamma_{\chi}\chi_t - \eta_{\chi}\chi^2\chi_t + F_{\chi}\sin(\omega_{\chi}t).$$

The execution record identifies **Simulation 20.5 FROZEN** as the baseline and **external sinusoidal forcing only** as the new component.

Objective

Simulation 20.6 asked:

Can the frozen nonlinear dissipative solver accept external work while maintaining a calculable balance among stored energy, linear loss, nonlinear loss, and forcing work?

The forcing parameters were

$$F_{\chi} = 0.002$$

and

$$\omega_{\chi} = 0.42.$$

All previously frozen coefficients were retained:

$$\begin{aligned}
c_\chi &= 1, \mu_\chi = 0.35, \\
\lambda_\chi &= 20, \gamma_\chi = 0.02, \\
\eta_\chi &= 80.
\end{aligned}$$

No constitutive analyzer was introduced, no physical reference scales were assigned, and no new manuscript mapping was assumed.

The Complete Dimensionless Solver Equation

By Simulation 20.6, the staged construction had reached

$$\begin{aligned}
\chi_{tt} = & c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3 \\
& - \gamma_\chi \chi_t - \eta_\chi \chi^2 \chi_t + F_\chi \sin(\omega_\chi t).
\end{aligned}$$

Each term had entered through a separate audit:

20.1	:	$c_\chi^2 \chi_{xx}$
20.2	:	$-\mu_\chi \chi$
20.3	:	$-\lambda_\chi \chi^3$
20.4A	:	$-\gamma_\chi \chi_t$
20.5	:	$-\eta_\chi \chi^2 \chi_t$
20.6	:	$+F_\chi \sin(\omega_\chi t)$

The complete equation had therefore been **assembled progressively rather than introduced as an untested whole**.

The Energy Law Now Included Input and Loss

With forcing present, the field was an open energy system.

Simulation 20.6 used the expected law

$$\frac{dE}{dt} = -P_{\text{linear}} - P_{\text{nonlinear}} + P_{\text{forcing}}.$$

Equivalently, the integrated balance was

$$E + L_{\text{linear}} + L_{\text{nonlinear}} - W_{\text{forcing}} = E_0.$$

These relationships are explicitly stated in the Simulation 20.6 execution record.

This was the most complete energy-accounting system constructed to this point.

The solver now tracked:

$$E(t)$$

instantaneous stored field energy,

$$L_{\text{linear}}(t)$$

cumulative linear damping loss,

$$L_{\text{nonlinear}}(t)$$

cumulative nonlinear damping loss,

and

$$W_{\text{forcing}}(t)$$

cumulative work supplied by the external forcing.

The balance quantity became

$$B(t) = E(t) + L_{\text{linear}}(t) + L_{\text{nonlinear}}(t) - W_{\text{forcing}}(t).$$

For correct accounting,

$$B(t) \approx E(0).$$

Numerical Configuration

The established baseline remained unchanged:

$$\begin{aligned}N &= 1024, \\L &= 1, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4},\end{aligned}$$

and

$$T_{\text{final}} = 80.$$

The Courant number remained

$$\boxed{\text{CFL} = 0.512}.$$

The solver therefore added external forcing without simultaneously altering the grid, timestep, propagation coefficient, restoring coefficient, nonlinear coefficient, or either damping coefficient.

Again:

$$\boxed{\text{Frozen Baseline} + \text{One New Physical Component.}}$$

External Work Was Calculated, Not Prescribed as Net Energy Gain

The forcing amplitude was fixed, but the **net work transferred to the field was not**.

At $t = 5$,

$$W_{\text{forcing}} = 5.123360 \times 10^{-6}.$$

At $t = 20$,

$$W_{\text{forcing}} = 5.985204 \times 10^{-5}.$$

At $t = 30$,

$$W_{\text{forcing}} = 2.937078 \times 10^{-5}.$$

At $t = 50$,

$$W_{\text{forcing}} = 5.251305 \times 10^{-5},$$

and at $t = 80$,

$$W_{\text{forcing}} = 4.658439585136 \times 10^{-5}.$$

The cumulative forcing-work history was therefore not monotonic.

That behavior is important. A sinusoidal driver does not automatically transfer positive energy into the field at every instant. The sign and magnitude of instantaneous work depend on the relationship between the forcing and the field velocity.

Thus,

$$\text{Applied Forcing} \neq \text{Prescribed Energy Increase.}$$

The energy transfer remained an output of the evolving system.

Final Energy Accounting

The initial total energy was

$$E_0 = 5.547677454538 \times 10^{-3}.$$

At $t = 80$, the remaining field energy was

$$E(80) = 6.389135960763 \times 10^{-4}.$$

The cumulative linear damping loss was

$$L_{\text{linear}} = 3.515110074301 \times 10^{-3},$$

while cumulative nonlinear damping removed

$$L_{\text{nonlinear}} = 1.440963200054 \times 10^{-3}.$$

The accumulated forcing work was

$$W_{\text{forcing}} = 4.658439585136 \times 10^{-5}.$$

The resulting compensated energy was

$$E_{\text{comp}} = 5.548402474580 \times 10^{-3}.$$

The final forcing-balance error was

$$1.306889320735 \times 10^{-4},$$

and the maximum reported balance error was

$$1.425652397040 \times 10^{-4}.$$

The solution remained finite and both the CFL and finite-solution tests passed.

Grid Refinement

Simulation 20.6 also tested the forced compensated balance at a common comparison time of

$$T = 10.$$

The three resolutions again preserved

$$\text{CFL} = 0.512.$$

The maximum balance errors were:

Grid	N	Maximum Balance Error
------	-----	-----------------------

Coarse	512	4.856585×10^{-4}
--------	-----	---------------------------

Grid	N	Maximum Balance Error
------	-----	-----------------------

Baseline	1024	1.256575×10^{-4}
----------	------	---------------------------

Fine	2048	3.135991×10^{-5}
------	------	---------------------------

The progression was

$$4.856585 \times 10^{-4} \rightarrow 1.256575 \times 10^{-4} \rightarrow 3.135991 \times 10^{-5}.$$

Each refinement reduced the balance residual by approximately a factor of four.

The forcing work at $T = 10$ was also essentially unchanged across the three resolutions:

$$1.976230 \times 10^{-5},$$

$$1.976229 \times 10^{-5},$$

$$1.976229 \times 10^{-5}.$$

This provided strong evidence that the externally driven response was converging rather than being created by grid-dependent numerical behavior.

What Simulation 20.6 Established

Simulation 20.6 completed the staged assembly of the dimensionless field solver used for the subsequent qualification program.

The system could now represent all of the mechanisms introduced during this sequence:

Propagation

Linear Restoring Response

Cubic Nonlinear Self-Interaction

Linear Dissipation

Nonlinear Dissipation

and

External Forcing.

More importantly, it contained explicit energy accounting for both directions of transfer:

External Work → Field Energy → Dissipative Loss.

The final state remained a calculated result.

No target amplitude, target energy, or desired endpoint had been programmed into the solver.

Simulation 20.6 Disposition

The Simulation 20.6 theory-control record confirms that all previously frozen terms were retained, external sinusoidal forcing was the only newly added physical term, and Simulation 20.5 remained unchanged.

The next simulation explicitly identifies its baseline as

Simulation 20.6 FROZEN.

The development sequence had therefore reached

20.1 → 20.1A → 20.2 → 20.3 → 20.4 → 20.4A → 20.5 → 20.6.

with

SIMULATION 20.6 — FROZEN

At this point, the central components of the dimensionless solver had been assembled.

The next question was no longer:

Can another physical term be added?

It was more fundamental:

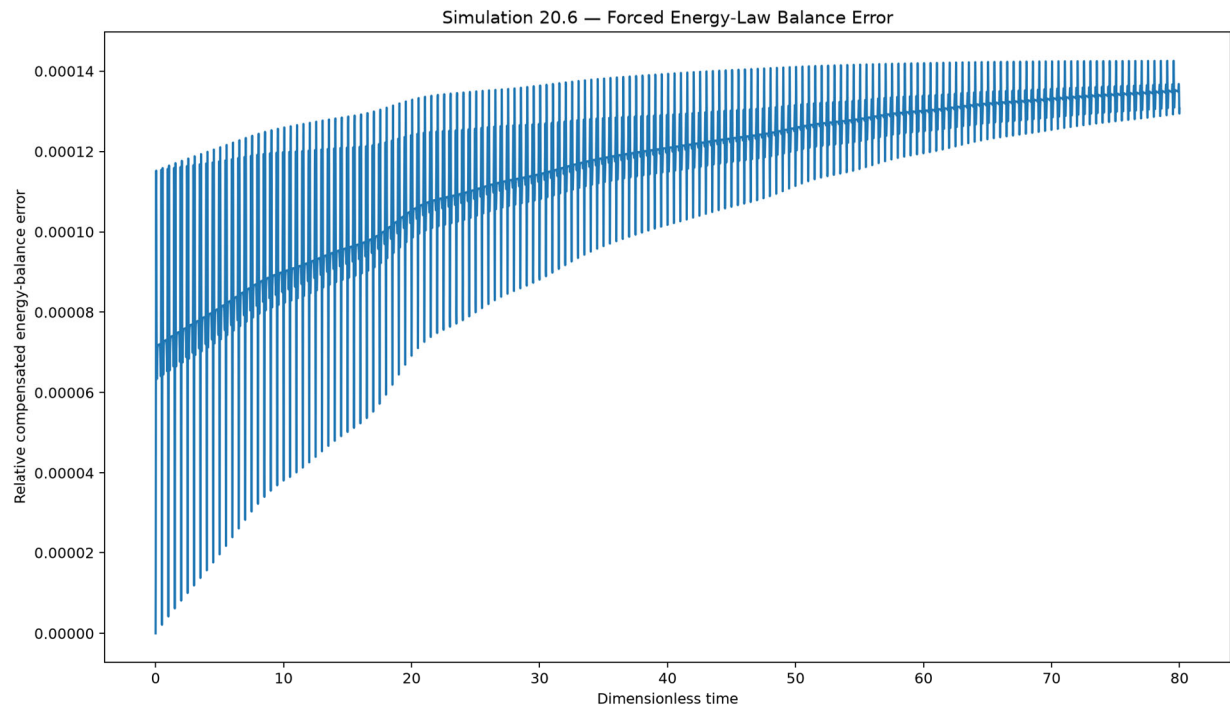
Does this dimensionless solver represent the same dynamics when arbitrary reference scales are changed?

That question began one of the most important audit sequences in the entire development record:

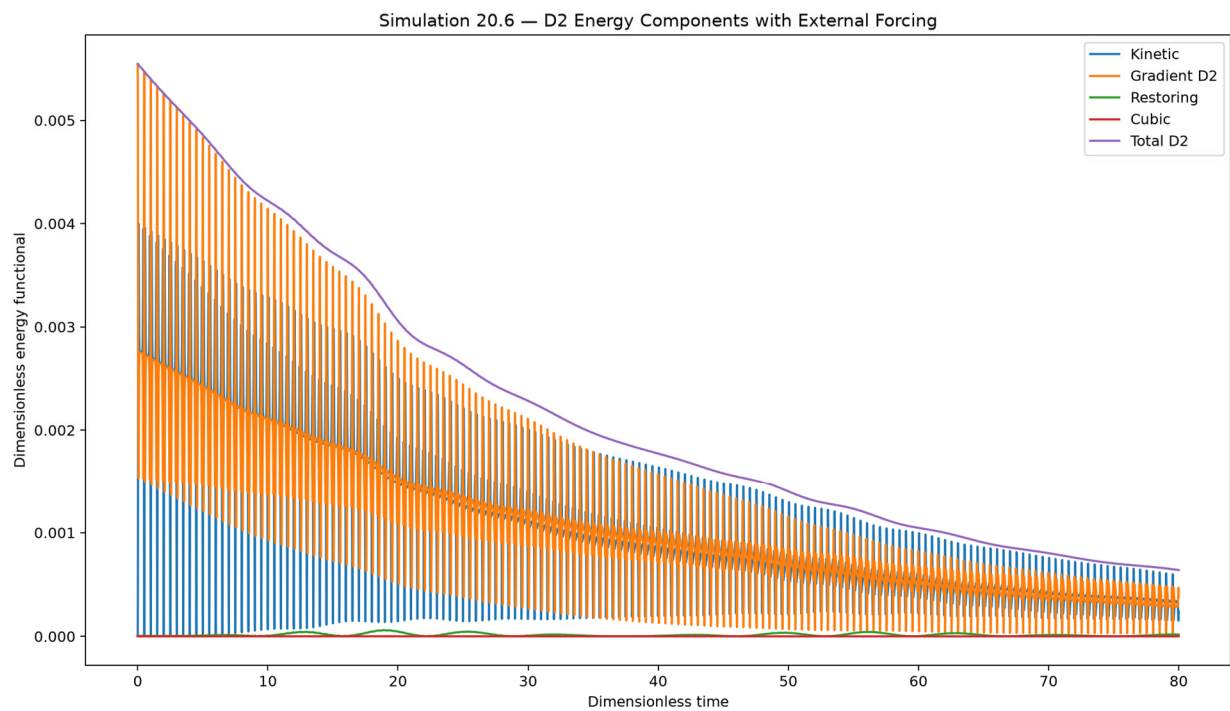
20.7 → 20.7A → 20.7B.

Unlike the preceding stages, this sequence added **no new physics**.
It attacked the dimensional architecture of the solver itself.

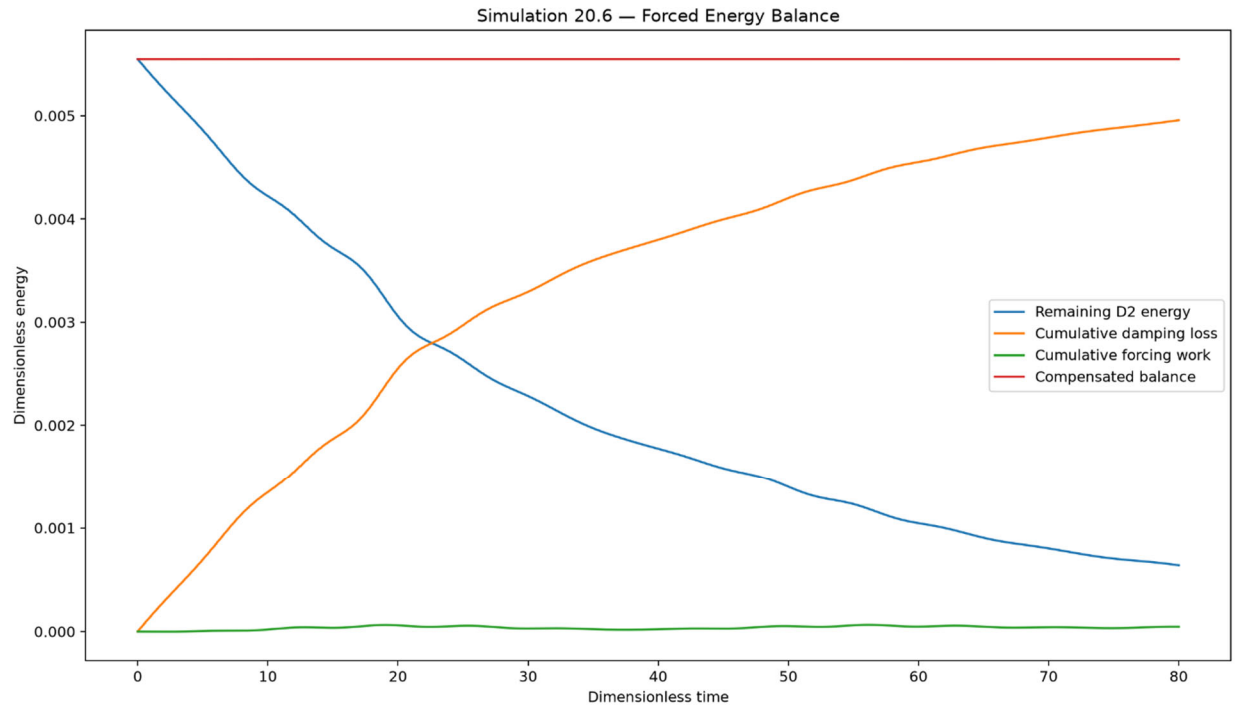
[FIGURE 014 — Simulation 20.6]



[FIGURE 015 — Simulation 20.6]



[FIGURE 016 — Simulation 20.6]



[FIGURE 017 — Simulation 20.6]

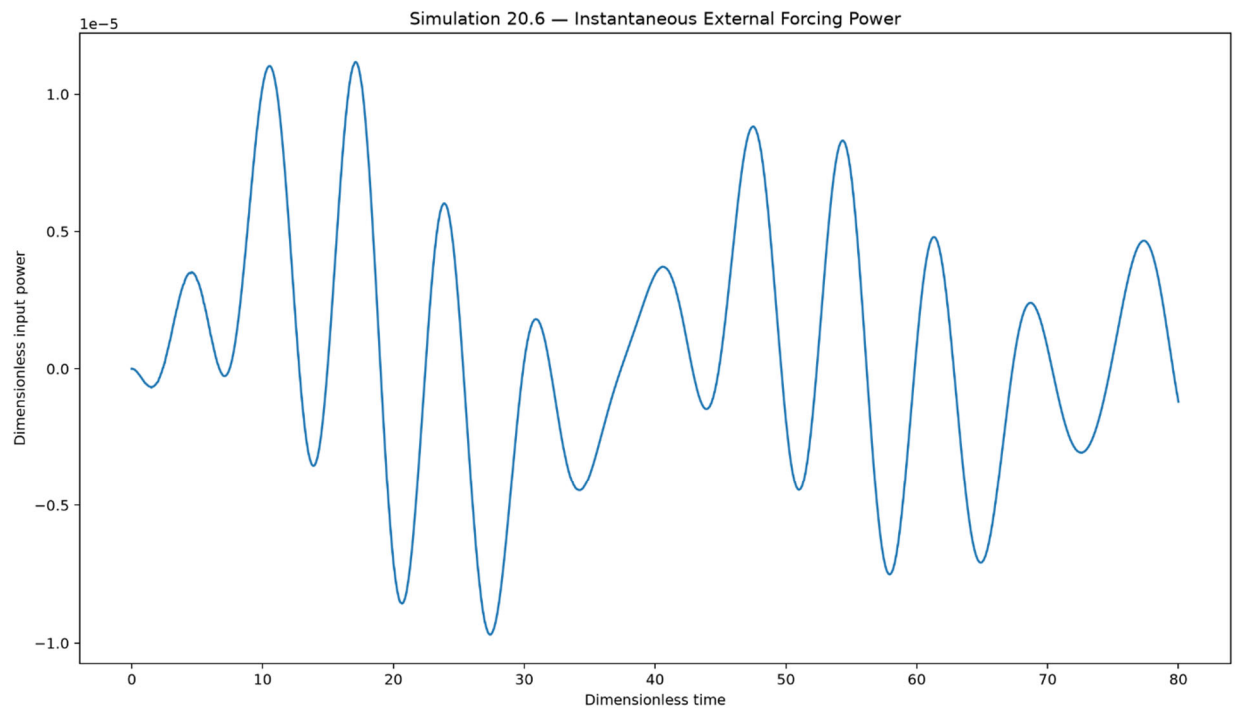


Figure 013-017. Simulation 20.6 — External Forcing Audit. External sinusoidal forcing was added to the frozen nonlinear dissipative solver. The calculation independently tracked stored field energy, linear damping loss, nonlinear damping loss, and external forcing work. Grid refinement reduced the forced compensated-balance residual from 4.856585×10^{-4} at $N = 512$ to 3.135991×10^{-5} at $N = 2048$, while the calculated forcing work at the common comparison time remained essentially invariant across resolution.

Simulation 20.7 — Dimensionless Scale-Invariance Audit

With Simulation 20.6 frozen, the staged construction of the dimensionless solver was complete. The next phase did **not** introduce another physical mechanism.

Instead, Simulation 20.7 attacked a more fundamental question:

Are the calculated dimensionless dynamics independent of the arbitrary reference scales used to express the corresponding dimensional quantities?

This was essential because the solver had intentionally been constructed without assigning a physical IAW length, time, or field-amplitude scale.

Simulation 20.7 therefore introduced

NO NEW PHYSICAL TERMS

and retained the complete frozen Simulation 20.6 equation:

$$\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3 - \gamma_\chi \chi_t - \eta_\chi \chi^2 \chi_t + F_\chi \sin(\omega_\chi t).$$

The execution record explicitly identifies Simulation 20.6 as frozen, states that no new physical terms were introduced, and identifies the reference scales as **audit scales only—not physical IAW scales**.

Objective

The objective was not to determine the physical size or timescale of the IAW.

It was to determine whether arbitrary choices of reference units altered the underlying dimensionless solution.

The distinction is fundamental:

Reference Scale $\neq$ Physical IAW Scale.

If two mathematically equivalent scale choices produced different dimensionless dynamics, the solver would contain an unwanted dependence on the representation used to describe the system.

Simulation 20.7 therefore tested the transformation chain

Dimensionless Coefficients → Dimensional Audit Representation → Recovered Dimensionless Coefficients

and then compared the resulting field histories, velocity histories, energy histories, losses, forcing work, and compensated balance.

Reference-Scale Cases

Three deliberately different scale combinations were used.

Case A

$$L_0 = 1, T_0 = 1, \Psi_0 = 1$$

Case B

$$L_0 = 10, T_0 = 2, \Psi_0 = 0.5$$

Case C

$$L_0 = 0.1, T_0 = 5, \Psi_0 = 4$$

These were not proposed black-hole scales.

They were deliberately varied numerical reference scales used to test the transformation architecture.

Forward and Inverse Dimensional Mapping

For the dimensionless field equation, the previously established mapping was

$$c_\chi = \frac{cT_0}{L_0},$$
$$\mu_\chi = aT_0^2,$$

$$\begin{aligned}
\lambda_\chi &= b\Psi_0^2 T_0^2, \\
\gamma_\chi &= gT_0, \\
\eta_\chi &= h\Psi_0^2 T_0, \\
F_\chi &= \frac{T_0^2}{\Psi_0} S_0,
\end{aligned}$$

and

$$\omega_\chi = \omega T_0.$$

The test was straightforward:

1. Begin with the frozen dimensionless coefficients.
2. Select L_0, T_0, Ψ_0 .
3. construct the corresponding dimensional audit coefficients.
4. invert the mapping.
5. recover the original dimensionless coefficients.
6. evolve the system.
7. compare the results.

The dimensional coefficients changed between cases, as they should.

The recovered dimensionless coefficients did not.

For example, Case B produced dimensional audit coefficients including

$$\begin{aligned}
c &= 5, \quad a = 0.0875, \\
b &= 20, \quad g = 0.01, \\
h &= 160,
\end{aligned}$$

yet recovered

$$\begin{aligned}
c_\chi &= 1, \quad \mu_\chi = 0.35, \\
\lambda_\chi &= 20, \quad \gamma_\chi = 0.02, \\
\eta_\chi &= 80, \quad F_\chi = 0.002, \quad \omega_\chi = 0.42.
\end{aligned}$$

This was exactly the behavior required of a correct invertible scale transformation.

Coefficient Round-Trip Test

Across the tested transformations, the maximum coefficient round-trip error was

$$5.551115123126 \times 10^{-17}.$$

This is effectively numerical roundoff for the calculation.

Thus, the coefficient mapping itself passed.

But coefficient recovery alone was not enough.

The solver still had to demonstrate that the **evolved solution** was invariant.

Field Equivalence Passed

The final fields from Cases A, B, and C were compared using relative L_2 error.

For A versus B,

$$\epsilon_{\chi}^{A/B} = 0.$$

For A versus C,

$$\epsilon_{\chi}^{A/C} = 2.433322111344 \times 10^{-14}.$$

For B versus C,

$$\epsilon_{\chi}^{B/C} = 2.433322111344 \times 10^{-14}.$$

The field-equivalence tests were therefore passed.

The arbitrary scale transformations had not materially altered the evolved field.

Energy and Balance Histories Also Passed

The scale-invariance audit extended beyond the final field.

The maximum energy-history relative error was

$$1.875893978223 \times 10^{-14}.$$

The maximum damping-loss relative error was

$$8.878495741495 \times 10^{-15}.$$

The maximum forcing-work relative error was

$$1.645231943043 \times 10^{-14}.$$

The maximum compensated-balance relative error was

$$1.907158858393 \times 10^{-14}.$$

These portions of the audit passed.

The solver was therefore reproducing essentially identical field and energy behavior under the tested reference-scale transformations.

But one test did not pass.

Velocity Equivalence Failed

The velocity comparison produced

$$\epsilon_{\dot{\chi}}^{A/B} = 0,$$

but

$$\epsilon_{\dot{\chi}}^{A/C} = 1.058851580773 \times 10^{-12}$$

and

$$\epsilon_{\chi}^{B/C} = 1.058851580773 \times 10^{-12}.$$

The programmed audit criterion classified this result as

FAIL.

Consequently, despite passing coefficient recovery, field equivalence, energy-history equivalence, damping-loss equivalence, forcing-work equivalence, compensated-balance equivalence, and finite-solution testing, the official result remained:

OVERALL SIMULATION 20.7 RESULT: FAIL

The actual audit record preserves that result.

This result was not overridden simply because the discrepancy was small.

That matters.

A failed programmed criterion remained a failure.

No Physics Was Changed

The failure did not justify altering the governing equation.

Simulation 20.7's theory-control record confirms that all frozen terms remained unchanged:

$$\begin{aligned} &-\mu_{\chi}\chi, \\ &-\lambda_{\chi}\chi^3, \\ &-\gamma_{\chi}\chi_t, \\ &-\eta_{\chi}\chi^2\chi_t, \end{aligned}$$

and

$$+F_{\chi}\sin(\omega_{\chi}t).$$

No physical terms were added. The reference scales remained audit scales only, no physical IAW scale was assigned, no constitutive analyzer was introduced, and Simulation 20.6 remained frozen.

The appropriate response was therefore not
change the physics,

but

investigate the failed audit.

Simulation 20.7 Disposition

Simulation 20.7 was **not frozen**.

Its disposition was

SIMULATION 20.7 — FAIL

with failure isolated to the velocity-equivalence criterion.

This initiated a corrective audit sequence:

20.7 → 20.7A.

And importantly, 20.7A would not be allowed to alter the underlying physics merely to obtain a passing result.

Simulation 20.7A — First Scale-Invariance Audit Correction

Simulation 20.7A returned to the failed scale-invariance audit while preserving the complete frozen Simulation 20.6 physical system.

Its execution record explicitly labels the revision:

20.7A status: Audit correction only

and again states:

New physical terms: NONE.

The same three reference-scale cases were retained:

$$(L_0, T_0, \Psi_0) = (1, 1, 1), \\ (10, 2, 0.5),$$

and

$$(0.1, 5, 4).$$

The test therefore remained an audit of the same mathematical system rather than a replacement experiment.

Velocity Equivalence Was Corrected

The revised audit treatment allowed the velocity-equivalence test to pass.

The underlying pairwise value remained

$$1.058851580773 \times 10^{-12}$$

for the A/C and B/C comparisons.

This is an important part of the development history.

The numerical solution had not suddenly changed.

The audit treatment of the comparison had.

Simulation 20.7A therefore resolved the first failure identified in 20.7.

But it exposed another.

Forcing-Work Equivalence Failed

After the first audit correction, the forcing-work comparison became the failing criterion.

The maximum forcing-work relative error was reported as

$$1.573867315895 \times 10^{-12}.$$

The other principal comparison errors remained extremely small:

$$\begin{aligned}\epsilon_{\chi}^{A/C} &= 2.433322111344 \times 10^{-14}, \\ \epsilon_{\dot{\chi}}^{A/C} &= 1.058851580773 \times 10^{-12}, \\ \epsilon_E &= 1.648325550094 \times 10^{-14}, \\ \epsilon_{\text{damping}} &= 4.781997887962 \times 10^{-15},\end{aligned}$$

and

$$\epsilon_{\text{balance}} = 6.253096364142 \times 10^{-15}.$$

Yet the forcing-work criterion failed.

Therefore:

OVERALL SIMULATION 20.7A RESULT: FAIL

The development program again did not declare success simply because nearly every other result passed.

Why the Second Failure Mattered

Forcing work is a particularly sensitive comparison because the accumulated work history can pass through or approach zero.

A conventional pointwise relative-error calculation has the form

$$\epsilon = \frac{|W_A - W_B|}{|W_{\text{reference}}|}.$$

If

$$W_{\text{reference}} \rightarrow 0,$$

then the denominator becomes very small.

A tiny absolute difference can therefore produce a comparatively large relative error even when the two physical histories remain nearly indistinguishable.

This was the issue that had to be addressed next.

But again, the correct response was not to change the forcing physics.

The comparison metric itself required further examination.

Simulation 20.7A Disposition

Simulation 20.7A corrected the first audit issue but did not complete the qualification.

Its disposition remained

SIMULATION 20.7A — FAIL

and the sequence continued:

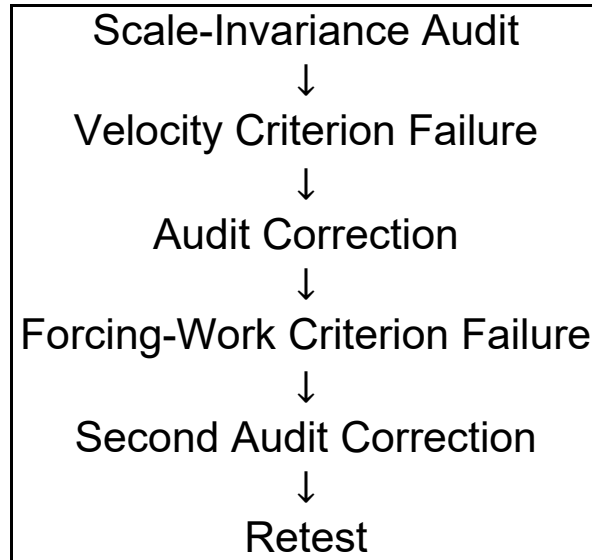
20.7 FAIL → 20.7A FAIL → 20.7B.

This is exactly why the lettered revisions belong in the manuscript.

The history was not:

scale test → pass.

It was:



The next revision, **Simulation 20.7B**, became the decisive test.

Simulation 20.7B — Final Dimensionless Scale-Invariance Audit

Simulation 20.7B again retained the complete frozen Simulation 20.6 physical system.

The record states:

- **20.7B status: Audit correction only**
- **New physical terms: NONE**
- Reference scales remained **audit scales only—not physical IAW scales**.

The same three scale cases were retained.

Nothing about the underlying physical equation was changed to manufacture a passing result.

Instead, the audit itself was brought into mathematical agreement with the discrete solver being tested.

Discrete Gradient-Energy Correction

One important correction concerned the gradient-energy diagnostic.

The final audit matched the energy diagnostic to the frozen periodic second-difference Laplacian.

The discrete relation used was

$$-\langle \chi, D_2 \chi \rangle = \sum_i \left(\frac{\chi_{i+1} - \chi_i}{\Delta x} \right)^2 \Delta x.$$

This removed a mismatch between the discrete operator evolving the field and the diagnostic used to judge its energy behavior.

The 20.7B theory-control record explicitly states that the energy-gradient diagnostic now matched the frozen discrete Laplacian.

Forcing-Work Comparison Near Zero

The forcing-work equivalence calculation was also corrected.

Rather than allowing a near-zero instantaneous value to dominate the relative-error denominator, 20.7B used a **global-history normalization** for the forcing-work comparison.

The physical forcing history was not altered.

Only the method used to compare equivalent histories was corrected.

The theory-control record explicitly identifies this change:

History equivalence uses global-history normalization near zero.

This directly addressed the failure exposed by Simulation 20.7A.

Active Single-Case Energy-Balance Test

Simulation 20.7B also made the individual energy-balance requirement an explicit PASS/FAIL criterion.

The tolerance was

$$\epsilon_{\text{balance}} \leq 1.0 \times 10^{-5}.$$

The maximum single-case balance error was

$$1.452066764256 \times 10^{-8}.$$

That is approximately three orders of magnitude below the programmed limit.

Final 20.7B Results

The complete audit passed.

The maximum coefficient round-trip error was

$$5.551115123126 \times 10^{-17}.$$

Field A/C equivalence error:

$$2.433322111344 \times 10^{-14}.$$

Velocity A/C equivalence error:

$$1.058851580773 \times 10^{-12}.$$

Maximum energy-history relative error:

$$3.439138960075 \times 10^{-15}.$$

Maximum damping-loss relative error:

$$4.725266482009 \times 10^{-15}.$$

Maximum forcing-work relative error:

$$1.582556440451 \times 10^{-14}.$$

Maximum compensated-balance relative error:

$$6.409304360174 \times 10^{-15}.$$

Maximum single-case balance error:

$$1.452066764256 \times 10^{-8}.$$

Every programmed test passed.

Therefore:

OVERALL SIMULATION 20.7B RESULT: PASS

What Simulation 20.7B Established

The result supports a precise conclusion:

The locked Simulation 20.6 dimensionless dynamics are invariant under the tested reference-scale transformations.

It does **not** establish a physical IAW scale. That distinction must remain explicit:

Scale Invariance Verified $\neq$ Physical IAW Scale Identified.

The arbitrary audit choices

$$L_0, T_0, \Psi_0$$

could be changed substantially while the recovered dimensionless dynamics remained equivalent. That was exactly what the dimensionless formulation required.

Simulation 20.7B Disposition

The full scale-invariance sequence was therefore:

20.7 FAIL $\rightarrow$ 20.7A FAIL $\rightarrow$ 20.7B PASS.

No physical term was altered during this sequence. The failures attacked the audit architecture, and the corrections remained diagnostic. The final disposition became

SIMULATION 20.7B — LOCKED

and the next simulation explicitly identifies its baseline as

Simulation 20.7B LOCKED

This was a major transition in the qualification program. The question was no longer whether arbitrary reference-scale choices changed the solution. That question had been answered. The next attack was purely numerical:

Does the complete frozen solver converge toward a common solution as the spatial and temporal resolution are increased?

That became:

[FIGURE INSERT — Simulations 20.7 / 20.7A / 20.7B]

This sequence deserves a combined figure or full-page validation plate because it shows the actual audit process exceptionally well.

I recommend:

1. **A/B/C final-field overlay** from 20.7B.
2. **Scale transformation / coefficient round-trip comparison.**
3. **Energy, damping-loss, forcing-work, and compensated-balance equivalence errors.**
4. A small three-stage result panel:

20.7 FAIL → 20.7A FAIL → 20.7B PASS

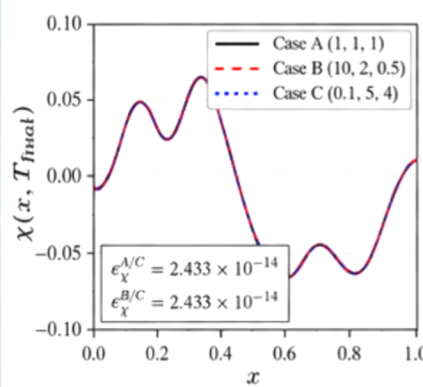
**Figure 018. Simulations 20.7 – 20.7A – 20.7B:
Dimensionless Scale-Invariance Audit Comparison**

Simulation 20.7 (Initial Test)		FAIL	Simulation 20.7A (First Correction)		FAIL	Simulation 20.7B (Final Audit)		PASS
Baseline	Simulation 20.6 (frozen)		Baseline	Simulation 20.6 (frozen)		Baseline	Simulation 20.6 (frozen)	
New physical terms	None		New physical terms	None		New physical terms	None	
Status	Initial scale-invariance test		Status	Audit correction only		Status	Audit correction only	
Reference scales	Audit scales only (not physical IAW scales) A: (1, 1, 1) B: (10, 2, 0.5) C: (0.1, 5, 4)		Reference scales	Audit scales only (not physical IAW scales) A: (1, 1, 1) B: (10, 2, 0.5) C: (0.1, 5, 4)		Reference scales	Audit scales only (not physical IAW scales) A: (1, 1, 1) B: (10, 2, 0.5) C: (0.1, 5, 4)	
Changes from 20.6	None (tests only)		Changes from 20.7	Audit treatment revised (velocity comparison corrected)		Changes from 20.7A	- Discrete gradient-energy diagnostic - Global-history normalization for forcing work near zero - Active single-case energy-balance test	

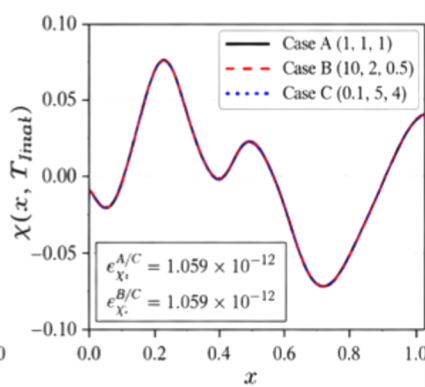
Comparison of Key Results

Test / Metric	20.7 Initial Test	20.7A First Correction	20.7B Final Audit
Coefficient round-trip error	5.551×10^{-17}	5.551×10^{-17}	5.551×10^{-17}
Field equivalence (A/C)	2.433×10^{-14}	2.433×10^{-14}	2.433×10^{-14}
Velocity equivalence (A/C)	1.059×10^{-12} FAIL	1.059×10^{-12} PASS	1.059×10^{-12} PASS
Energy history error	1.876×10^{-14} PASS	1.648×10^{-14} PASS	3.439×10^{-15} PASS
Damping-loss error	8.878×10^{-15} PASS	4.782×10^{-15} PASS	4.725×10^{-15} PASS
Forcing-work error	1.645×10^{-14} PASS	1.574×10^{-12} FAIL	1.583×10^{-14} PASS
Compensated-balance error	1.907×10^{-14} PASS	6.253×10^{-15} PASS	6.409×10^{-15} PASS
Max single-case balance error	—	—	1.452×10^{-8} (tolerance 1.0×10^{-5})
Overall result	FAIL	FAIL	PASS

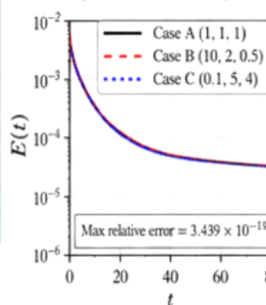
Final Field Comparison (A, B, C) (Simulation 20.7B)



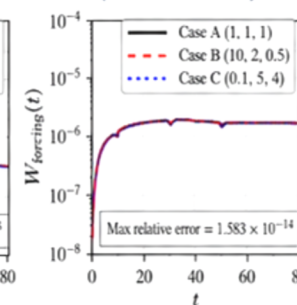
Velocity Comparison (A, B, C) (Simulation 20.7B)



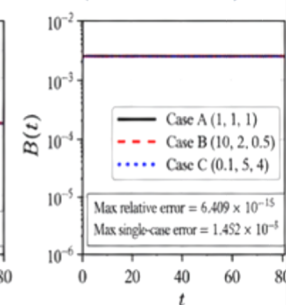
Energy History Comparison (Simulation 20.7B)



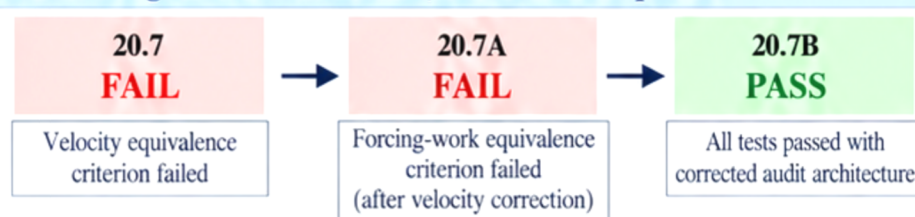
Forcing Work History Comparison (Simulation 20.7B)



Compensated Energy Balance (Simulation 20.7B)



Audit Progression and Failure/Correction Sequence



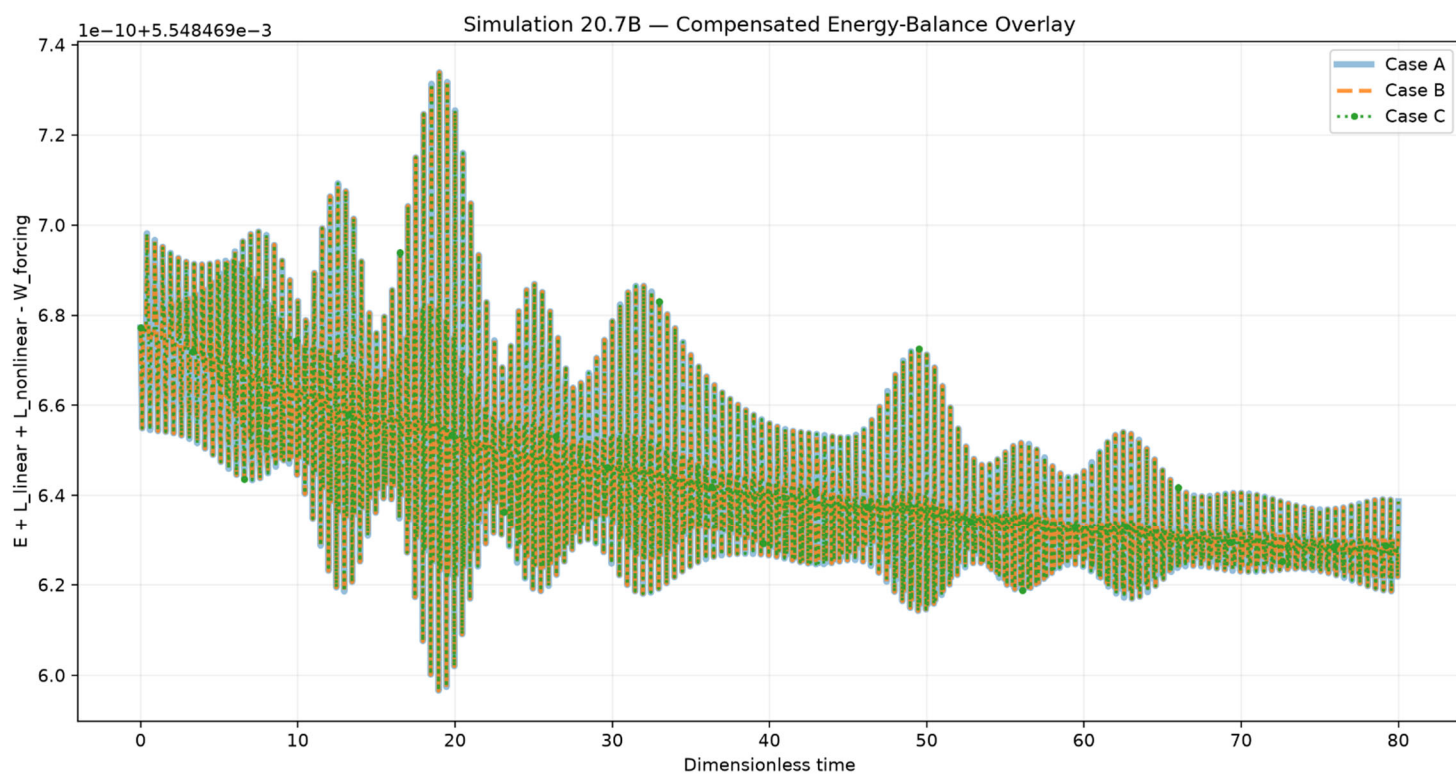
Key Corrections in 20.7B

- Matched gradient-energy diagnostic to frozen discrete Laplacian.
- Global-history normalization for forcing-work comparison near zero.
- Active single-case energy-balance test (tolerance 1.0×10^{-5}).
- No new physical terms introduced.
- Reference scales remain audit scales only (not physical IAW scales).
- Frozen Simulation 20.6 physics retained.

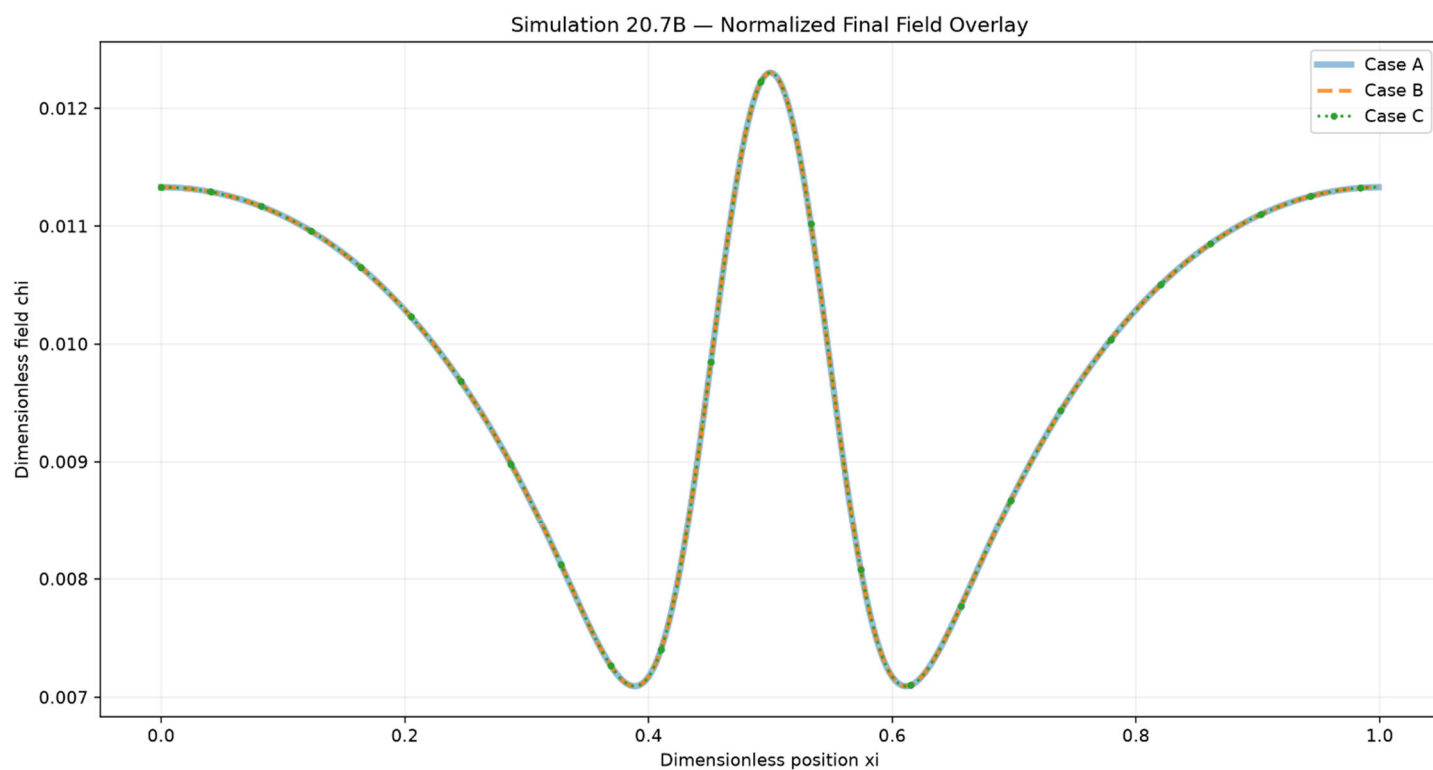
Development Sequence and Final Disposition



[FIGURE 019 — Simulations 20.7 / 20.7A / 20.7B]



[FIGURE 020 — Simulations 20.7 / 20.7A / 20.7B]



[FIGURE 021 — Simulations 20.7 / 20.7A / 20.7B]

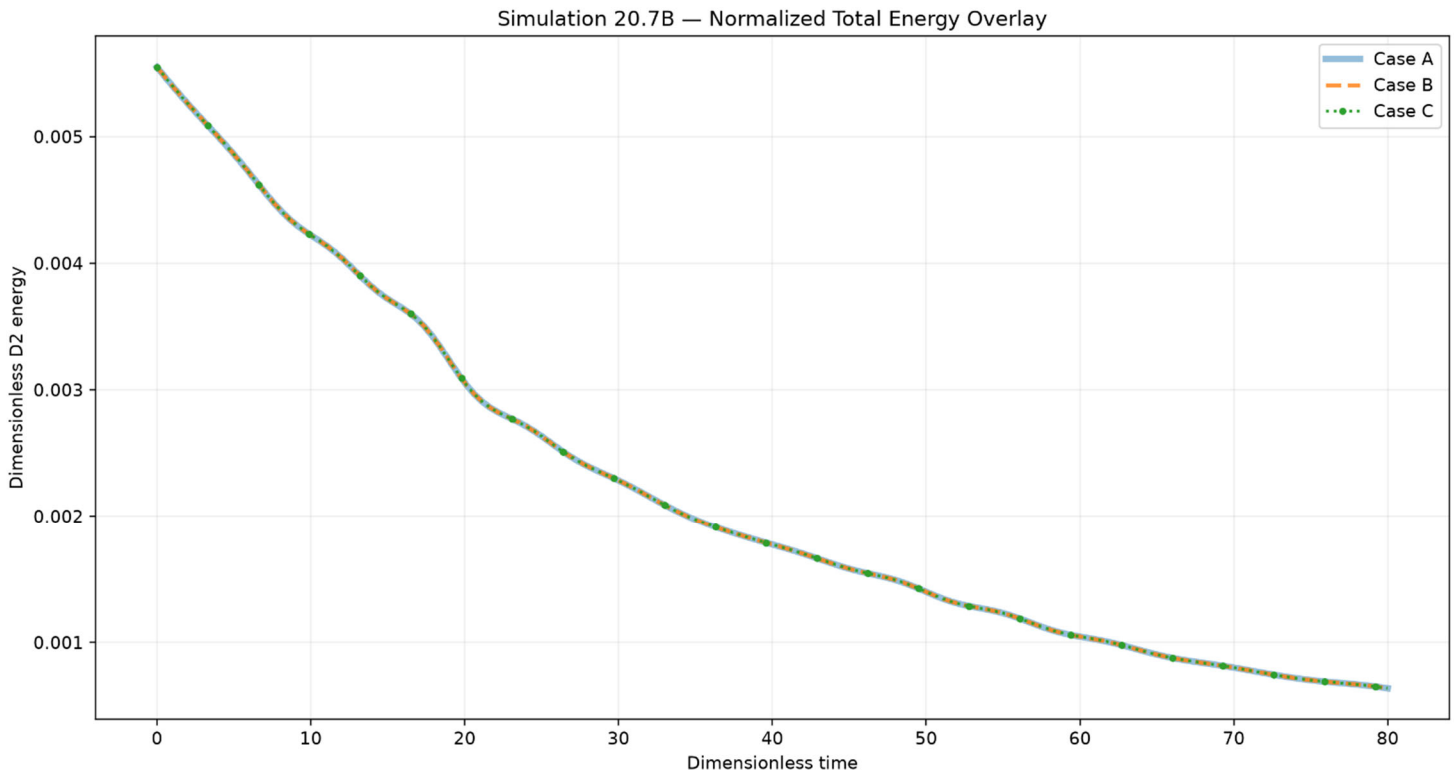


Figure 017-020. Simulations 20.7–20.7B — Dimensionless Scale-Invariance Qualification. Three substantially different reference-scale combinations were applied to the frozen Simulation 20.6 architecture. Simulation 20.7 failed the velocity-equivalence criterion, and Simulation 20.7A subsequently exposed a forcing-work comparison problem near zero. Both revisions preserved the governing physics. Simulation 20.7B corrected the audit architecture, including discrete gradient-energy consistency and global-history normalization for near-zero forcing-work comparisons. The final audit passed all twelve programmed tests, establishing invariance of the frozen dimensionless dynamics under the tested reference-scale transformations.

Simulation 20.8 — Numerical Resolution and Convergence Audit

With Simulation 20.7B LOCKED, the solver passed the dimensionless scale-invariance audit. The next test attacked a different potential weakness:

Does the complete LOCKED solution converge as numerical resolution is increased? Simulation 20.8 introduced

NO NEW PHYSICAL TERMS

and

NO CONSTITUTIVE CHANGES.

Its sole purpose was numerical resolution and convergence testing. The execution record explicitly identifies **Simulation 20.7B LOCKED** as the baseline and states that no physical, constitutive, or reference-scale changes were introduced.

LOCKED Solver

The complete frozen dimensionless equation remained

$$\chi_{tt} = c_\chi^2 \chi_{xx} - \mu_\chi \chi - \lambda_\chi \chi^3 - \gamma_\chi \chi_t - \eta_\chi \chi^2 \chi_t + F_\chi \sin(\omega_\chi t).$$

The coefficients remained

$$\begin{aligned} c_\chi &= 1, \mu_\chi = 0.35, \\ \lambda_\chi &= 20, \gamma_\chi = 0.02, \\ \eta_\chi &= 80, F_\chi = 0.002, \\ \omega_\chi &= 0.42. \end{aligned}$$

Nothing in the governing system was adjusted to improve convergence. Only numerical resolutions changed.

Three-Level Resolution Test

Three grids were evaluated.

R1 — Coarse

$$\begin{aligned}N &= 512, \\ \Delta x &= 1.953125 \times 10^{-3}, \\ \Delta t &= 1.0 \times 10^{-3}.\end{aligned}$$

R2 — Baseline

$$\begin{aligned}N &= 1024, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4}.\end{aligned}$$

R3 — Fine

$$\begin{aligned}N &= 2048, \\ \Delta x &= 4.8828125 \times 10^{-4}, \\ \Delta t &= 2.5 \times 10^{-4}.\end{aligned}$$

In every case,

$$\boxed{\text{CFL} = 0.512}.$$

Thus, spatial and temporal resolution were refined together while the Courant number remained fixed. This was essential because the audit needed to isolate **resolution dependence**, not simultaneously introduce a different numerical stability condition.

Final Energy Convergence

The calculated final energies were

$$\begin{aligned}E_{512} &= 5.668297502035 \times 10^{-4}, \\ E_{1024} &= 5.665182170394 \times 10^{-4},\end{aligned}$$

and

$$E_{2048} = 5.664201728574 \times 10^{-4}.$$

The progression

$$5.6682975 \times 10^{-4} \rightarrow 5.6651822 \times 10^{-4} \rightarrow 5.6642017 \times 10^{-4}$$

shows the final energy approaching a common value as the grid was refined. The calculation was not moving away from a solution. It was converging toward one.

Energy-Balance Error Improved with Resolution

The maximum compensated energy-balance errors were:

$$1.500719038175 \times 10^{-7}$$

for $N = 512$,

$$1.796814866482 \times 10^{-8}$$

for $N = 1024$,

and

$$4.179644819006 \times 10^{-9}$$

for $N = 2048$.

Therefore,

$$1.50072 \times 10^{-7} \rightarrow 1.79681 \times 10^{-8} \rightarrow 4.17964 \times 10^{-9}.$$

The finest grid reduced the maximum balance residual by approximately a factor of 36 relative to the coarse grid. This provided an independent numerical confirmation of the energy-accounting architecture established in the preceding simulations.

Field Convergence

The relative field differences between successive resolutions were

$$e_{12} = 7.168308596583 \times 10^{-3}$$

for the 512/1024 comparison and

$$e_{23} = 2.030630562918 \times 10^{-3}$$

for the 1024/2048 comparison. The error decreased substantially:

$$e_{23} < e_{12}.$$

The observed field convergence order was

$$p_{\chi} = 1.81970496.$$

This value was calculated from the results. It was not imposed as an acceptance target.

Velocity Convergence

The corresponding velocity errors were

$$e_{12,v} = 2.446174351308 \times 10^{-1}$$

and

$$e_{23,v} = 6.083405032566 \times 10^{-2}.$$

Again,

$$e_{23,v} < e_{12,v}.$$

The observed velocity convergence order was

$$p_v = 2.00757627.$$

The velocity calculation therefore displayed approximately second-order convergence over the tested resolutions.

Internal Histories Converged as Well

Simulation 20.8 did not limit the comparison to the final field. The histories of individual energy and loss components were also compared between successive resolutions. The maximum common-time errors decreased from coarse/baseline to baseline/fine.

Examples included:

Total energy: $4.938629698966 \times 10^{-4} \rightarrow 1.180072802888 \times 10^{-4}$,

Kinetic energy: $9.575030320125 \times 10^{-3} \rightarrow 3.816459354715 \times 10^{-3}$,

Gradient energy: $6.026321378190 \times 10^{-3} \rightarrow 2.391285322960 \times 10^{-3}$,

Restoring energy: $4.582075092633 \times 10^{-4} \rightarrow 1.369292344419 \times 10^{-4}$,

and

Cubic energy: $3.839362958989 \times 10^{-3} \rightarrow 1.009823007561 \times 10^{-3}$.

Thus, the convergence was not confined to one convenient output quantity. The internal energetic decomposition was moving toward common histories as resolution increased.

Dissipation and Forcing Histories Converged

The same behavior occurred in the energy-transfer channels. Linear damping-loss history error decreased from

$2.725876775019 \times 10^{-4}$

to

$6.410803182915 \times 10^{-5}$.

Nonlinear damping-loss history error decreased from

$$2.709881392249 \times 10^{-3}$$

to

$$6.466138477067 \times 10^{-4}.$$

Forcing-work history error decreased from

$$2.513722078228 \times 10^{-6}$$

to

$$6.127774544103 \times 10^{-7}.$$

The compensated-balance history error decreased from

$$1.116413867594 \times 10^{-4}$$

to

$$2.788962707464 \times 10^{-5}.$$

The refinement therefore improved not only the field solution but also the calculated energy-transfer histories.

Why Simulation 20.8 Was a Stronger Test Than Simply Running a Finer Grid

A single high-resolution calculation does not establish convergence. The important comparison is

Coarse → Baseline → Fine.

The solution must demonstrate that the difference between successive resolutions decreases.

Simulation 20.8 showed precisely that:

$$\|\chi_{1024} - \chi_{2048}\| < \|\chi_{512} - \chi_{1024}\|$$

and

$$\|\dot{\chi}_{1024} - \dot{\chi}_{2048}\| < \|\dot{\chi}_{512} - \dot{\chi}_{1024}\|.$$

At the same time,

$$\epsilon_{\text{balance},2048} < \epsilon_{\text{balance},1024} < \epsilon_{\text{balance},512}.$$

These are substantially stronger numerical statements than simply reporting that a 2048-point calculation completed successfully.

What Simulation 20.8 Established

Simulation 20.8 established four important properties of the LOCKED SOLVER over the tested resolution range:

1. **The field solution converged.**
2. **The velocity solution converged.**
3. **The internal energy and transfer histories moved toward common solutions.**
4. **The compensated energy-balance residual decreased with refinement.**

The observed convergence orders were

$$p_{\chi} = 1.81970496$$

and

$$p_v = 2.00757627.$$

No physical term was modified to obtain those results. No constitutive coefficient was tuned between resolutions. No physical IAW scale was introduced.

Simulation 20.8 Disposition

All programmed Simulation 20.8 audit checks passed:

- LOCKED 20.7B physics unchanged;
- finite solution at all three resolutions;
- identical CFL;
- decreasing field error;
- decreasing velocity error;
- improved fine-grid energy balance.

Therefore,

OVERALL SIMULATION 20.8 RESULT: PASS

and the disposition became

SIMULATION 20.8 — LOCKED.

The solver had now survived two fundamentally different attacks:

Reference-Scale Transformation

and

Numerical Resolution Refinement.

But both tests operated around the same frozen physical coefficient set. That left another question:

Would the solver remain numerically coherent if the physical coefficients were deliberately pushed away from the baseline?

Simulation 20.9 was designed to answer exactly that.

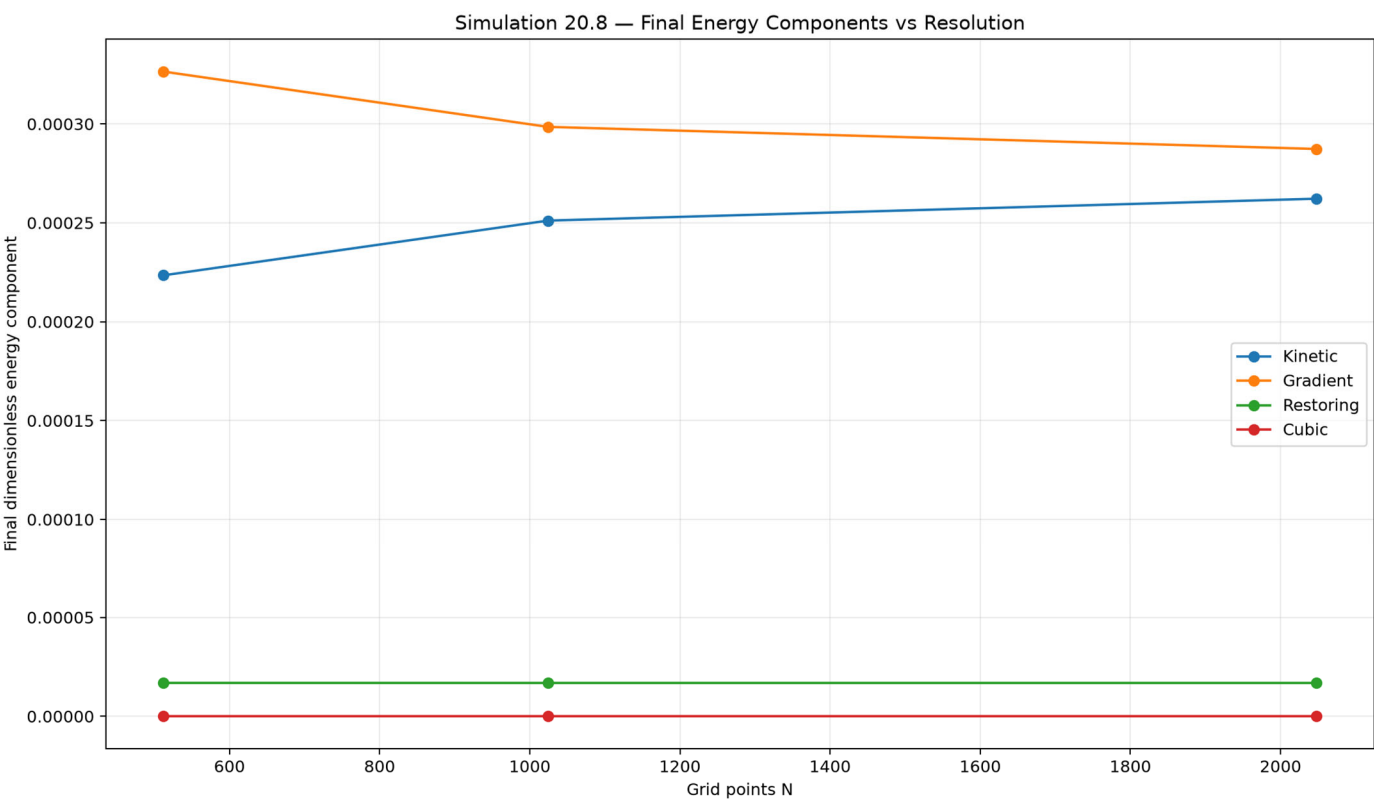
Simulation 20.9 — Parameter Stress-Test Matrix

Instead of changing one component and then freezing it, Simulation 20.9 would deliberately attack the completed solver with multiple parameter perturbations—

including a combined stress case—and allow the calculated responses to determine whether the architecture remained stable.

[FIGURE 022 — Simulation 20.8]

Figure A — Resolution Convergence



[FIGURE 022 — Simulation 20.8]

Figure B — Energy-Balance Convergence

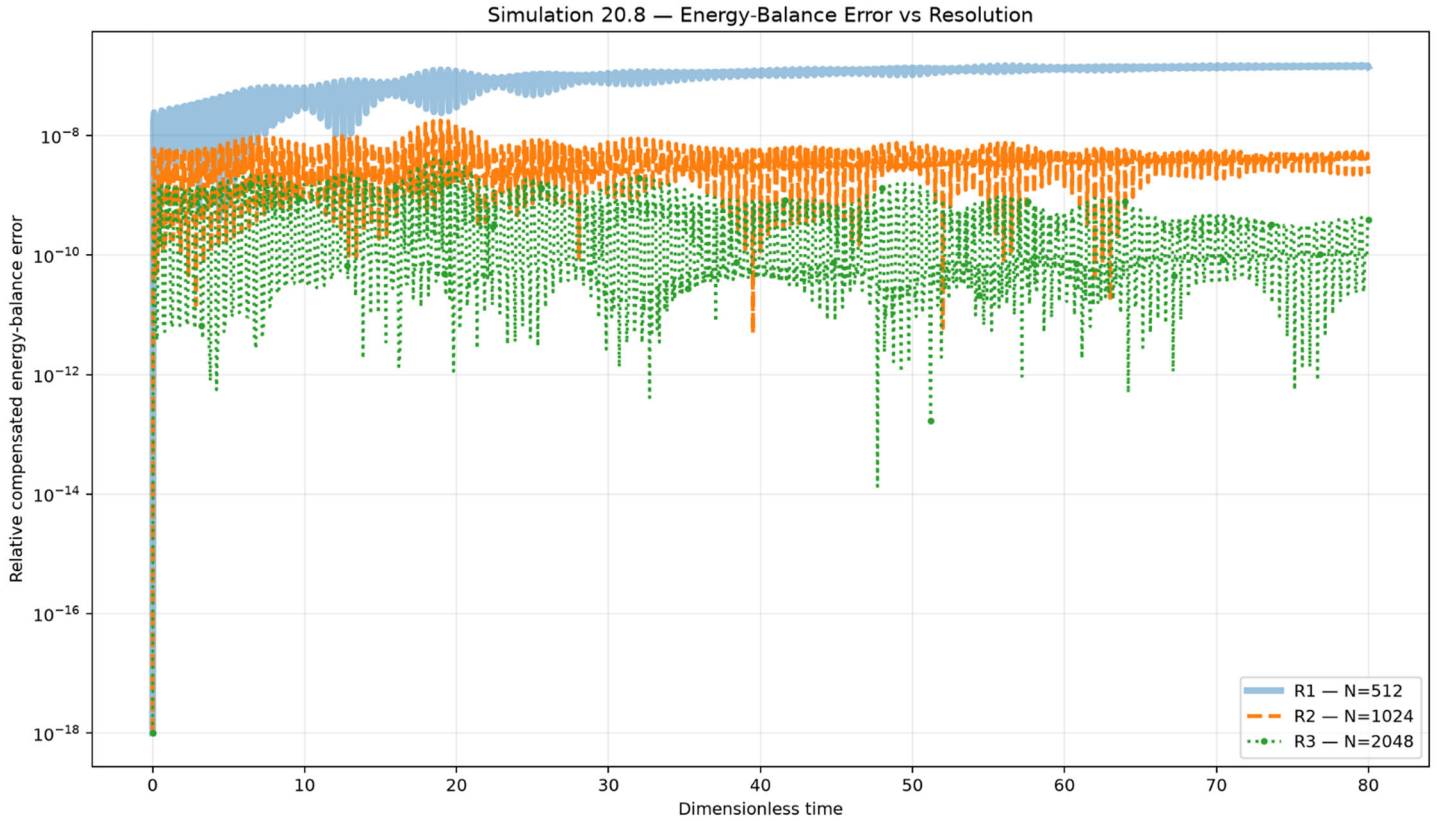


Figure 021-022. Simulation 20.8 — Numerical Resolution and Convergence Audit.

The LOCKED Simulation 20.7B solver was evaluated at three spatial and temporal resolutions while maintaining a constant CFL of 0.512. Successive refinement reduced both field and velocity differences, producing observed convergence orders of $p_\chi = 1.81970$ and $p_v = 2.00758$. The maximum compensated energy-balance residual simultaneously decreased from 1.500719×10^{-7} at $N = 512$ to 4.179645×10^{-9} at $N = 2048$, demonstrating numerical convergence of the frozen solver without modification of its physical coefficients.

Simulation 20.9 — Parameter Stress-Test Matrix

Simulation 20.8 established that the frozen solver converged under numerical refinement. Simulation 20.9 then attacked a different question:

Does the completed solver remain numerically coherent when its physical coefficients are deliberately driven away from the baseline?

Unlike the earlier construction sequence, Simulation 20.9 introduced **no new governing term**. Instead, existing coefficients were intentionally perturbed individually and in combination.

The purpose was not to make the solver reproduce a preferred response.

It was to see what the solver actually did.

Change the inputs → Calculate the response → Audit the result.

This directly implemented the central computational rule of the development program:

The endpoint is an output, not an input.

Objective

By this stage the dimensionless equation was

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi - \lambda_{\chi} \chi^3 - \gamma_{\chi} \chi_t - \eta_{\chi} \chi^2 \chi_t + F_{\chi} \sin(\omega_{\chi} t).$$

Previous simulations had tested the individual terms and then subjected the completed solver to scale-invariance and resolution-convergence audits.

Simulation 20.9 now asked three related questions:

1. Does the solution remain finite when coefficients are stressed?
2. Does the energy-accounting architecture remain controlled?
3. Does changing different coefficients produce physically differentiated responses rather than essentially identical numerical outputs?

The third question is particularly important.

A solver that remains numerically stable but responds almost identically to materially different coefficient changes would itself require investigation.

Baseline Stress-Control Case

The control case, S0, retained the established coefficients:

$$c_\chi = 1$$

$$\mu_\chi = 0.35$$

$$\lambda_\chi = 20$$

$$\gamma_\chi = 0.02$$

$$\eta_\chi = 80$$

$$F_\chi = 0.002$$

and

$$\omega_\chi = 0.42.$$

The numerical baseline remained

$$N = 1024, \Delta x = 9.765625 \times 10^{-4}, \\ \Delta t = 5 \times 10^{-4}, \text{ CFL} = 0.512.$$

Simulation 20.9 therefore began from the already qualified baseline before intentionally perturbing selected coefficients.

Seven-Case Stress Matrix

Seven cases were evaluated.

Case	Stress condition
S0	Baseline control
S1	Cubic coefficient doubled: $\lambda_\chi = 40$
S2	Linear damping increased $\times 4$: $\gamma_\chi = 0.08$
S3	Nonlinear damping doubled: $\eta_\chi = 160$
S4	Forcing amplitude increased $\times 4$: $F_\chi = 0.008$
S5	Forcing frequency changed: $\omega_\chi = 0.47$
S6	Combined stress: $\lambda_\chi = 40, \eta_\chi = 160, F_\chi = 0.008, \omega_\chi = 0.47$

For S6,

$$\gamma_\chi = 0.02$$

remained unchanged.

This matrix attacked the solver through different parts of the governing equation rather than merely scaling every coefficient together.

Baseline S0 Response

The baseline calculation produced

$$\max |\chi| = 3.353126083850 \times 10^{-2},$$

and

$$\max |\chi_t| = 2.680625340718 \times 10^{-1}.$$

The final energy was

$$E_f = 5.665182170394 \times 10^{-4}.$$

Cumulative linear damping loss was

$$L_{\text{lin}} = 3.115559611221 \times 10^{-3},$$

while nonlinear damping loss was

$$L_{\text{nl}} = 1.270659976406 \times 10^{-3}.$$

The forcing work was

$$W = 4.605687201182 \times 10^{-5}.$$

Most importantly, the maximum compensated-balance error was

$$1.796814813450 \times 10^{-8}.$$

S0 passed.

This established the reference against which the stressed cases could be interpreted.

S1 — Cubic Self-Interaction Doubled

The first perturbation increased

$$\lambda_\chi: 20 \rightarrow 40.$$

The resulting maximum field amplitude was

$$\max |\chi| = 3.319538626186 \times 10^{-2},$$

with final energy

$$E_f = 5.794782539967 \times 10^{-4}.$$

The maximum balance error remained

$$1.777386776010 \times 10^{-8}.$$

S1 passed.

The important result was not simply that the calculation remained stable. Doubling the nonlinear self-interaction changed the calculated response without destroying the energy accounting.

S2 — Linear Damping Increased Fourfold

The second stress case changed

$$\gamma_\chi: 0.02 \rightarrow 0.08.$$

This produced one of the strongest responses in the entire matrix.

The maximum field amplitude fell to

$$\max |\chi| = 2.500000000000 \times 10^{-2},$$

and the final energy dropped dramatically to

$$E_f = 2.459798538213 \times 10^{-5}.$$

The cumulative linear damping loss increased to

$$L_{\text{lin}} = 4.602950219209 \times 10^{-3}.$$

The maximum compensated-balance error remained only

$$2.983104679942 \times 10^{-8}.$$

S2 passed.

This was a clear differentiated response:

$$\gamma_\chi \uparrow \Rightarrow \text{stronger linear dissipation and substantially lower final energy.}$$

The solver did not merely remain stable—it responded in the direction dictated by the modified term.

S3 — Nonlinear Damping Doubled

Next,

$$\eta_{\chi}: 80 \rightarrow 160.$$

The maximum field amplitude became

$$3.193294032490 \times 10^{-2},$$

and the final energy was

$$3.381646170114 \times 10^{-4}.$$

The nonlinear damping loss increased to

$$L_{\text{nl}} = 2.041517133237 \times 10^{-3}.$$

The maximum balance error was

$$2.409771112060 \times 10^{-8}.$$

S3 passed.

Again, the response was mechanism-specific.

Increasing the nonlinear damping coefficient increased the calculated nonlinear loss channel.

S4 — Forcing Amplitude Increased Fourfold

The external forcing amplitude was then changed from

$$F_{\chi} = 0.002$$

to

$$F_{\chi} = 0.008.$$

The maximum field amplitude increased substantially:

$$\max |\chi| = 5.866635646500 \times 10^{-2}.$$

The cumulative forcing work increased to

$$W = 1.008466573839 \times 10^{-3}.$$

For comparison, the S0 forcing work was only

$$4.605687201182 \times 10^{-5}.$$

The system therefore did not treat the stronger driver as an inert coefficient change.

It calculated a substantially different driven response.

The maximum balance error remained

$$4.196848498437 \times 10^{-8}.$$

S4 passed.

S5 — Forcing-Frequency Stress

The forcing amplitude returned to the baseline value while the forcing frequency changed from

$$0.42$$

to

$$0.47.$$

The resulting maximum field amplitude became

$$3.634098082307 \times 10^{-2},$$

with final energy

$$4.179697259002 \times 10^{-4}.$$

The cumulative forcing work increased relative to baseline to

$$1.134644869140 \times 10^{-4}.$$

The maximum balance error was

$$2.077303262773 \times 10^{-8}.$$

S5 passed.

The response demonstrated that changing forcing frequency altered the calculated work exchange and field evolution even when forcing amplitude remained fixed.

S6 — Combined Stress

The most demanding case simultaneously imposed

$$\begin{aligned}\lambda_{\chi} &= 40, \\ \eta_{\chi} &= 160, \\ F_{\chi} &= 0.008,\end{aligned}$$

and

$$\omega_{\chi} = 0.47.$$

Linear damping remained

$$\gamma_{\chi} = 0.02.$$

The resulting maximum field amplitude was

$$5.582215703232 \times 10^{-2}.$$

Maximum velocity remained finite at

$$2.678610936405 \times 10^{-1}.$$

Final energy was

$$1.888219262002 \times 10^{-4}.$$

The accumulated nonlinear damping loss increased to

$$5.420372895340 \times 10^{-3},$$

while forcing work reached

$$1.850266518548 \times 10^{-3}.$$

Despite simultaneous nonlinear, damping, amplitude, and frequency perturbations, the maximum compensated-balance error remained

$$5.872648077003 \times 10^{-8}.$$

S6 passed.

Comparative Stress Results

The principal outputs can be summarized as follows:

Case	$\max \chi $	Final E	Max Balance Error
S0	3.35313×10^{-2}	5.66518×10^{-4}	1.79681×10^{-8}
S1	3.31954×10^{-2}	5.79478×10^{-4}	1.77739×10^{-8}
S2	2.50000×10^{-2}	2.45980×10^{-5}	2.98310×10^{-8}
S3	3.19329×10^{-2}	3.38165×10^{-4}	2.40977×10^{-8}
S4	5.86664×10^{-2}	2.65818×10^{-4}	4.19685×10^{-8}
S5	3.63410×10^{-2}	4.17970×10^{-4}	2.07730×10^{-8}
S6	5.58222×10^{-2}	1.88822×10^{-4}	5.87265×10^{-8}

The stress matrix therefore did **not** produce seven essentially identical responses.
Instead:

Different coefficient perturbations → Different calculated dynamics.

Energy Accounting Survived Every Stress Case

All seven cases remained well within the established balance threshold

$$\epsilon_{\text{balance}} \leq 10^{-5}.$$

Even the largest residual, from S6,

$$5.872648077003 \times 10^{-8},$$

was approximately

170

times smaller than the acceptance limit. This is one of the strongest results of Simulation 20.9. The solver was not merely stable around one carefully selected baseline. Its energy accounting remained controlled while individual coefficients and multiple coefficients simultaneously were pushed away from that baseline.

What Simulation 20.9 Established

Simulation 20.9 demonstrated three properties simultaneously.

First:

Numerical robustness

All seven cases remained finite and passed their energy-balance tests.

Second:

Parameter sensitivity

Different coefficient perturbations produced materially different field amplitudes, final energies, damping losses, and forcing-work histories.

Third:

Mechanism differentiation

The response followed the mechanism being altered:

$$\begin{aligned}\gamma_{\chi} \uparrow &\rightarrow L_{\text{linear}} \uparrow, \\ \eta_{\chi} \uparrow &\rightarrow L_{\text{nonlinear}} \uparrow,\end{aligned}$$

and

$$F_{\chi} \uparrow \rightarrow W_{\text{forcing}} \uparrow \text{ and } |\chi|_{\text{max}} \uparrow.$$

This is exactly what a useful computational solver must do.

It must neither collapse when the inputs change nor ignore those changes.

No Endpoint Was Programmed

Simulation 20.9 is also one of the clearest demonstrations of the governing computational philosophy. The code did not contain instructions such as

$$E_f = E_{\text{expected}},$$

or

$$|\chi|_{\text{max}} = |\chi|_{\text{expected}}.$$

The coefficients were changed. The governing equation was evolved. The resulting amplitudes, velocities, energy losses, forcing work, and final energies were calculated.

Thus:

Input Parameters → Numerical Evolution → Calculated Endpoint.

Not:

Desired Endpoint → Adjusted Calculation.

That distinction is central to the scientific purpose of the Simulation 20 program.

Simulation 20.9 Disposition

All seven stress cases passed. The combined stress case remained finite and maintained a maximum compensated-balance residual of only

$$5.872648077003 \times 10^{-8}.$$

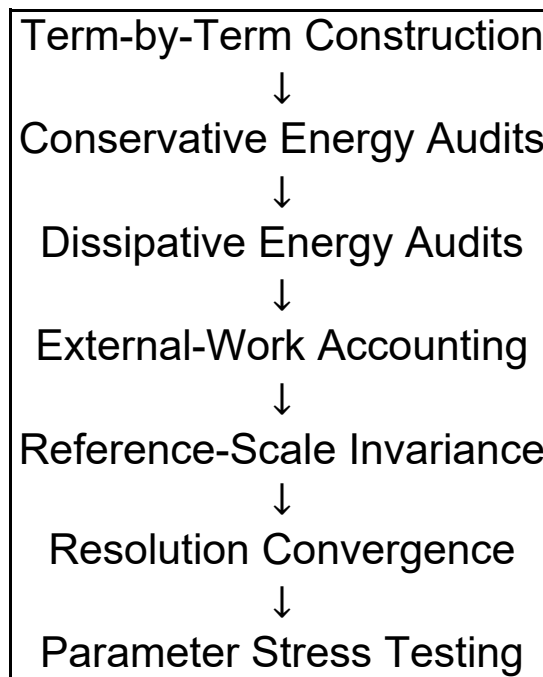
Therefore,

OVERALL SIMULATION 20.9 RESULT: PASS

and its disposition became

SIMULATION 20.9 — FROZEN.

At this point, the solver had passed:



One test remained.

Rather than introduce another physical mechanism, Simulation 20.10 would take the already developed solver and subject it to a **final integrated qualification**. That simulation would independently reproduce the baseline, repeat the full stress matrix, execute a fine-grid calculation, compare the baseline and refined solutions, and determine whether the numerical engine as a whole was ready to be frozen and locked.

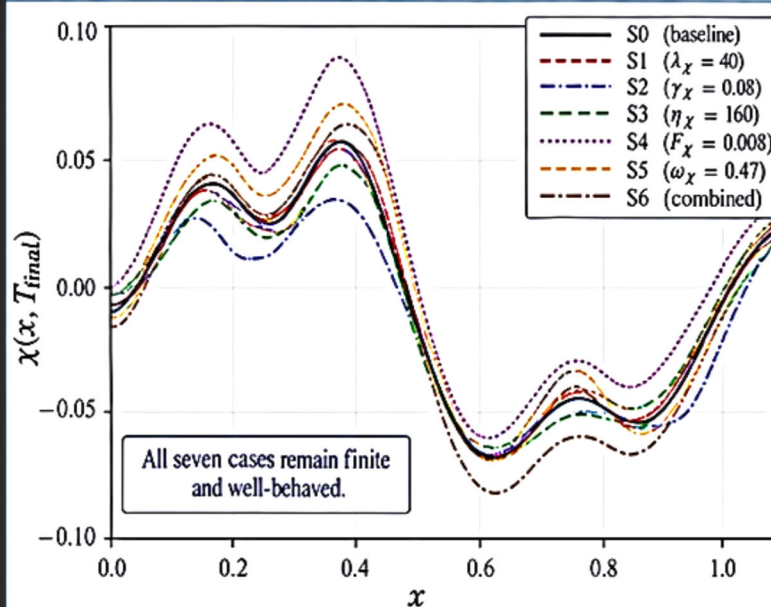
Simulation 20.10 — Final Integrated Solver Qualification
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Figure 023. Simulation 20.09 — Parameter Stress-Test Matrix
Seven coefficient stress cases for the LOCKED Simulation 20 solver.

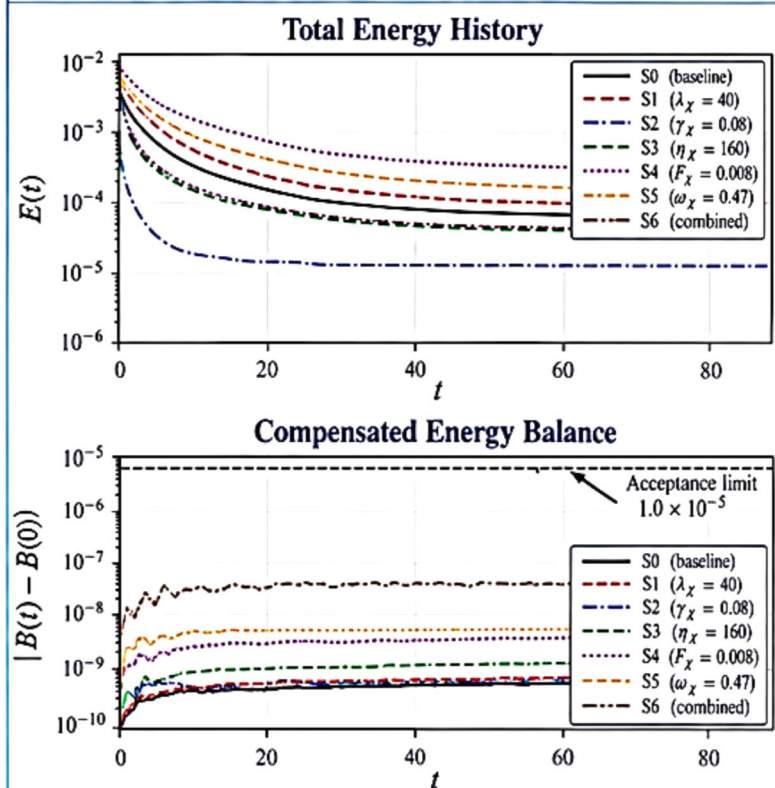
A. Parameter Stress-Test Matrix (Simulation 20.09)

Case	Stress Condition (Change from baseline)	Modified Parameter(s)	$\max \chi $	Final Energy E_f	Max Balance Error	Result
S0	Baseline control	None (baseline)	3.35313×10^{-2}	5.66518×10^{-4}	1.79681×10^{-8}	PASS
S1	Cubic coefficient doubled	$\lambda_x = 40$	3.31954×10^{-2}	5.79478×10^{-4}	1.77739×10^{-8}	PASS
S2	Linear damping $\times 4$	$\gamma_x = 0.08$	2.50000×10^{-2}	2.45980×10^{-5}	2.98310×10^{-8}	PASS
S3	Nonlinear damping $\times 2$	$\eta_x = 160$	3.19329×10^{-2}	3.38165×10^{-4}	2.40977×10^{-8}	PASS
S4	Forcing amplitude $\times 4$	$F_x = 0.008$	5.86664×10^{-2}	2.65818×10^{-4}	4.19685×10^{-8}	PASS
S5	Forcing frequency change	$\omega_x = 0.47$	3.63410×10^{-2}	4.17970×10^{-4}	2.07730×10^{-8}	PASS
S6	Combined stress	$\lambda_x = 40, \eta_x = 160$ $F_x = 0.008, \omega_x = 0.47$	5.58222×10^{-2}	1.88822×10^{-4}	5.87265×10^{-8}	PASS

B. Field Responses for All Stress Cases (Simulation 20.09)



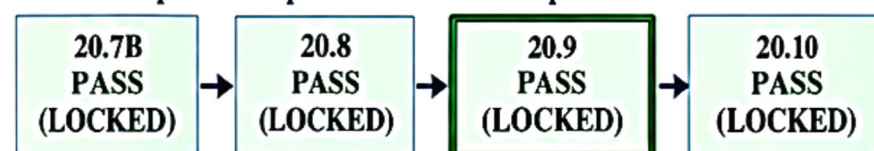
C. Energy Responses for All Stress Cases (Simulation 20.09)



D. Key Results Summary (Simulation 20.09)

Case	$\max \chi $	Final Energy E_f	Max Balance Error
S0	3.35313×10^{-2}	5.66518×10^{-4}	1.79681×10^{-8}
S1	3.31954×10^{-2}	5.79478×10^{-4}	1.77739×10^{-8}
S2	2.50000×10^{-2}	2.45980×10^{-5}	2.98310×10^{-8}
S3	3.19329×10^{-2}	3.38165×10^{-4}	2.40977×10^{-8}
S4	5.86664×10^{-2}	2.65818×10^{-4}	4.19685×10^{-8}
S5	3.63410×10^{-2}	4.17970×10^{-4}	2.07730×10^{-8}
S6	5.58222×10^{-2}	1.88822×10^{-4}	5.87265×10^{-8}

E. Development Sequence and Final Disposition



Key Conclusions (Simulation 20.09)

- All seven stress cases (S0–S6) passed.
- Each coefficient perturbation produced a distinct response.
- Energy accounting remained controlled for all cases.
- Maximum compensated-balance error: 5.87265×10^{-8} ($\approx 170\times$ below 1.0×10^{-5} acceptance limit).
- Solver ready for final integrated qualification (Simulation 20.10).
- Solver LOCKED as the qualified numerical engine.

Figure 023. Simulation 20.9 — Parameter Stress-Test Matrix. The locked solver was subjected to seven coefficient configurations ranging from the baseline control to simultaneous nonlinear, damping, forcing-amplitude, and forcing-frequency perturbations. Each case produced a distinct calculated response while remaining finite and satisfying the compensated energy-balance criterion. The most demanding combined stress case produced a maximum balance residual of 5.87265×10^{-8} , approximately 170 times below the 10^{-5} acceptance threshold.

Simulation 20.10 — Final Integrated Solver Qualification

Simulation 20.10 represented the final qualification stage of the numerical solver developed through the Simulation 20 program. No new physical mechanism was introduced. No constitutive coefficient was added. No physical reference scale was assigned. No target endpoint was specified. Instead, the previously developed numerical system was subjected to an integrated qualification containing three major components:

Independent Baseline Reproduction

Full Parameter Stress-Matrix Reproduction

and

Independent Fine-Grid Verification.

At the time of execution, Simulation 20.10 was deliberately identified as

CANDIDATE — NOT FROZEN.

Its status would be determined by the results. Only after those results were reviewed was the solver locked.

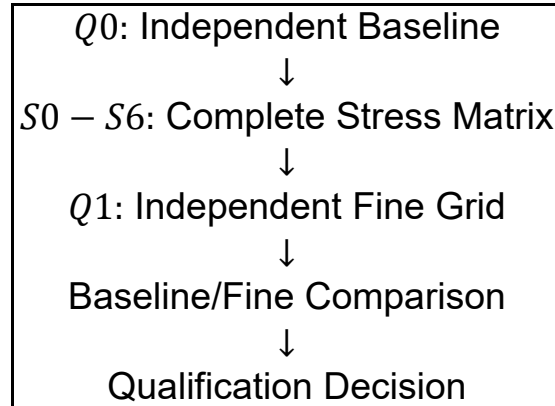
Objective

The purpose of Simulation 20.10 was different from the preceding simulations. The question was no longer whether an individual physical term worked.

It was:

Does the complete numerical engine reproduce the previously qualified baseline and stress responses, retain controlled energy accounting, and remain consistent under independent grid refinement when evaluated as a single integrated qualification program?

The qualification architecture was



The solver had to pass the complete sequence.

Locked Governing System

The equation remained

$$\chi_{tt} = c_{\chi}^2 \chi_{xx} - \mu_{\chi} \chi - \lambda_{\chi} \chi^3 - \gamma_{\chi} \chi_t - \eta_{\chi} \chi^2 \chi_t + F_{\chi} \sin(\omega_{\chi} t).$$

The baseline coefficients remained

$$\begin{aligned} c_{\chi} &= 1, \mu_{\chi} = 0.35, \\ \lambda_{\chi} &= 20, \gamma_{\chi} = 0.02, \\ \eta_{\chi} &= 80, F_{\chi} = 0.002, \\ \omega_{\chi} &= 0.42. \end{aligned}$$

The compensated energy law remained

$$B(t) = E(t) + L_{\text{lin}}(t) + L_{\text{nl}}(t) - W(t).$$

The qualification limit remained

$$\epsilon_{\text{balance}} \leq 10^{-5}.$$

Simulation 20.10 did not change these rules after seeing the results.

Q0 — Independent Baseline Reproduction

The first qualification case independently reproduced the baseline at

$$\begin{aligned}N &= 1024, \\ \Delta x &= 9.765625 \times 10^{-4}, \\ \Delta t &= 5.0 \times 10^{-4},\end{aligned}$$

With

$$\boxed{\text{CFL} = 0.512}.$$

The calculated maximum field amplitude was

$$\boxed{\max |\chi| = 3.353126083850 \times 10^{-2}}.$$

Maximum velocity was

$$\boxed{\max |\chi_t| = 2.680625340718 \times 10^{-1}}.$$

Maximum energy was

$$4.906680943866 \times 10^{-3},$$

while final energy was

$$\boxed{E_f = 5.665182170394 \times 10^{-4}}.$$

The cumulative energy transfers were

$$\begin{aligned}L_{\text{lin}} &= 3.115559611221 \times 10^{-3}, \\ L_{\text{nl}} &= 1.270659976406 \times 10^{-3},\end{aligned}$$

and

$$W = 4.605687201182 \times 10^{-5}.$$

The maximum compensation-balance error was

$$1.796814813450 \times 10^{-8}.$$

Therefore:

Q0 — PASS

The independently executed qualification baseline reproduced the previously established solver behavior.

Reproduction of the Complete Simulation 20.9 Stress Matrix

Simulation 20.10 then repeat all seven parameter cases from Simulation 20.9. This was important because qualification did not rely solely on the baseline. The solver had to reproduce its previously observed responses when subjected again to

- baseline conditions;
- doubled cubic self-interaction;
- fourfold linear damping;
- doubled nonlinear damping;
- fourfold forcing amplitude;
- forcing-frequency change;
- combined stress.

The resulting maximum balance errors were:

Case	Maximum Balance Error	Result
S0	$1.796814813450 \times 10^{-8}$	PASS
S1	$1.777386776010 \times 10^{-8}$	PASS
S2	$2.983104679942 \times 10^{-8}$	PASS
S3	$2.409771112060 \times 10^{-8}$	PASS
S4	$4.196848498437 \times 10^{-8}$	PASS
S5	$2.077303262773 \times 10^{-8}$	PASS
S6	$5.872648077003 \times 10^{-8}$	PASS

Thus,

$$S0, S1, S2, S3, S4, S5, S6 = \text{PASS.}$$

More importantly, these were the same differentiated responses established during Simulation 20.9. The qualification program therefore reproduced the prior stress-test behavior rather than merely generating another collection of stable solutions.

Q1 — Independent Fine-Grid Qualification

The solver was then executed independently at the finer resolution

$$N = 2048.$$

The spatial interval became

$$\Delta x = 4.8828125 \times 10^{-4},$$

and the timestep became

$$\Delta t = 2.5 \times 10^{-4}.$$

The CFL remained

$$0.512.$$

Thus, the qualification repeated the refinement strategy previously tested in Simulation 20.8 without changing the governing physics. The fine-grid calculation produced

$$\max |\chi| = 3.354708533054 \times 10^{-2},$$

and

$$\max |\chi_t| = 2.680625688132 \times 10^{-1}.$$

Maximum energy was

$$4.906817725528 \times 10^{-3},$$

and final energy was

$$E_f = 5.664201728574 \times 10^{-4}.$$

Cumulative linear damping loss was

$$3.115360696970 \times 10^{-3},$$

nonlinear damping loss was

$$1.271093691487 \times 10^{-3},$$

and forcing work was

$$4.605683388592 \times 10^{-5}.$$

The maximum compensated-balance error was

$$4.179646763439 \times 10^{-9}.$$

Therefore:

Q1 — PASS

Energy Accounting Improved on the Fine Grid

The Q0 baseline-grid balance error was

$$1.796814813450 \times 10^{-8}.$$

The Q1 fine-grid balance error was

$$4.179646763439 \times 10^{-9}.$$

Therefore,

$$1.79681 \times 10^{-8} \rightarrow 4.17965 \times 10^{-9}.$$

The fine-grid residual was approximately **4.3 times smaller** than the baseline-grid residual. This was an important independent confirmation of the convergence behavior previously observed in Simulation 20.8. The integrated qualification therefore did not merely reproduce the baseline. It reproduced the expected improvement under numerical refinement.

Baseline-to-Fine Field Comparison

Simulation 20.10 directly compared the final baseline and fine-grid states.

The relative field difference was

$$\epsilon_{\chi} = 2.030630562921 \times 10^{-3}.$$

The relative velocity difference was

$$\epsilon_{\dot{\chi}} = 6.083405032566 \times 10^{-2}.$$

The relative final-energy difference was

$$\epsilon_E = 1.730944389438 \times 10^{-4}.$$

These results independently reproduced the baseline-to-fine behavior established during the 1024 → 2048 comparison in Simulation 20.8. That agreement connected two separate qualification stages:

Simulation 20.8 Convergence Audit ↔ Simulation 20.10 Integrated Qualification.

Qualification Matrix

The final qualification matrix was therefore:

Qualification Test	Result
Q0 independent baseline	PASS
S0 baseline stress control	PASS
S1 cubic stress	PASS
S2 linear-damping stress	PASS
S3 nonlinear-damping stress	PASS
S4 forcing-amplitude stress	PASS
S5 forcing-frequency stress	PASS
S6 combined stress	PASS
Q1 independent fine grid	PASS
Fine-grid balance improves/holds	PASS

Thus:

OVERALL SIMULATION 20.10 RESULT: PASS

At execution, the correct conclusion was:

SOLVER QUALIFICATION CANDIDATE — PASS

The program itself did not automatically declare the architecture LOCKED. That remained a separate governance decision following review of the results.

No Corrective Simulation Was Required

Previous stages demonstrated that a failed audit would not be ignored. Simulation 20.7 produced a failure. Simulation 20.7A produced another failure. Both were retained in the development record, investigated, and corrected before 20.7B was accepted.

Simulation 20.10 did not produce a comparable unresolved numerical failure. There was therefore no numerical basis for constructing a corrective

20.10A.

The qualification program had done what it was designed to do. This is significant because the development sequence was not required to end at 20.10. Had the qualification failed,

20.10 → Failure Analysis → 20.10A

would have been entirely acceptable. The result determined the stopping point. The stopping point did not determine the result.

What Simulation 20.10 Established

Simulation 20.10 integrated the previous numerical validation stages into one final qualification. It established that the solver could:

Reproduce its frozen baseline

Reproduce the complete stress-response matrix

Remain finite under all qualification cases

Maintain controlled compensated energy accounting

Reproduce fine-grid convergence behavior

and

Calculate its endpoints without endpoint targeting.

The solver was no longer merely a sequence of individually successful numerical experiments. It had become a repeatedly tested numerical engine.

Final Lock

After the complete Simulation 20.10 results were reviewed, the numerical solver was explicitly frozen and locked. The final qualification chain became

20.7B FROZEN → 20.8 FROZEN → 20.9 FROZEN → 20.10 FROZEN + LOCKED.

This distinction matters. The code did not begin Simulation 20.10 with a predetermined declaration that the solver was final. At execution it remained

CANDIDATE — NOT FROZEN.

Only after

$Q0 = \text{PASS},$
 $S0 - S6 = \text{PASS},$
 $Q1 = \text{PASS},$

and

fine-grid verification = PASS

was the numerical solver accepted. The final disposition is therefore:

SIMULATION 20.10 — FROZEN AND LOCKED

What “Locked” Means

The freeze applies to the **qualified numerical solver architecture through Simulation 20.10.**

It does not mean that every unresolved theoretical question in the IAW has been answered. It does not automatically convert every computational term into an approved

constitutive term of the final physical theory. In particular, the previously identified distinction concerning

$$-\eta_{\chi}\chi^2\chi_t$$

remains.

The nonlinear damping term belongs to the frozen computational qualification sequence, but its relationship to the final approved IAW governing equation remains a separate theory-level decision.

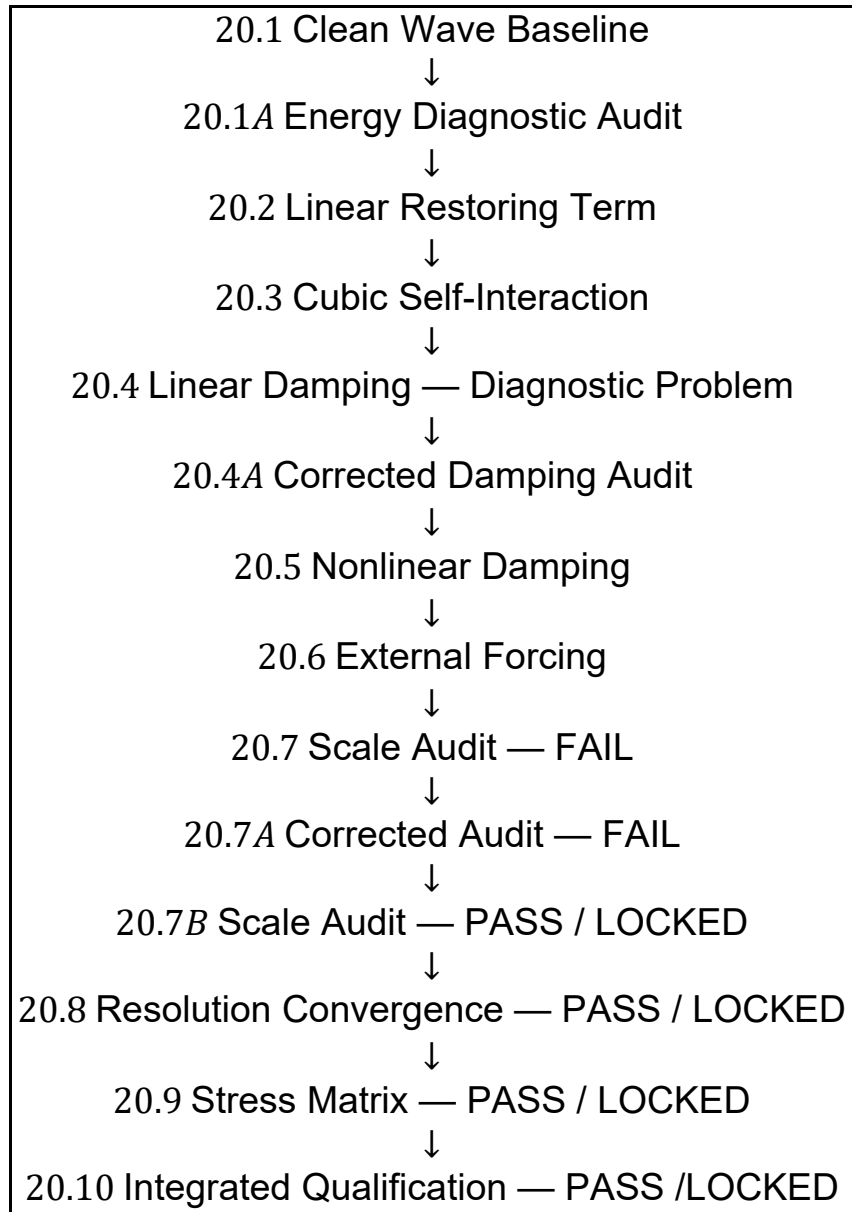
Therefore:

Numerical Solver Qualification $\neq$ Final Constitutive Closure.

That boundary prevents a successful numerical audit from being mistaken for authorization to alter the physical theory.

Completion of the Simulation 20 Solver Program

The complete development sequence can now be represented as



That is the computational development record. The failures remain part of it. The corrections remain part of it. The intermediate versions remain part of it. And the final solver exists because those earlier stages were preserved and audited rather than erased.

Everything is preserved. Only the presentation is selective.

Transition — From Solver Qualification to the ICSO

Simulation 20.10 completes the numerical-solver qualification portion of **Building the ICSO**. The next stage should **not** be Simulation 20.11 simply for the sake of continuing the numbering sequence. The numerical engine has reached a defensible stopping point. The next scientific task is to return to the operator that motivated this development:

$$\mathcal{I}: \mathcal{S}_{\text{rel}} \rightarrow \mathcal{C}_{\text{IAW}}$$

with

$$\mathcal{C}_{\text{IAW}} = \{\rho_{\text{eff}}, K^{ij}, \mu^2, \lambda, \gamma, S\}.$$

The qualified solver tells us that the resulting structural dynamics can be evolved and numerically interrogated. The remaining problem is deeper:

How does the relativistic physical state actually generate those constitutive quantities?

That is where **building the numerical solver ends and building the ICSO itself resumes**.

[FIGURE 024 — Simulation 20.10]

A — Qualification Matrix

$Q0, S0 - S6, Q1$

all visibly marked **PASS**.

B — Baseline vs Fine-Grid Field

final $N = 1024$ and $N = 2048$ solutions and identify

$$\epsilon_{\chi} = 2.03063 \times 10^{-3}.$$

C — Compensated Energy Balance

$$1.79681 \times 10^{-8} \rightarrow 4.17965 \times 10^{-9}.$$

D — Final Development Chain

20.7B LOCKED → 20.8 LOCKED → 20.9 LOCKED → 20.10 LOCKED.

Proposed caption:

Figure 024. Simulation 20.10 — Final Integrated Solver Qualification. The complete numerical engine independently reproduced the frozen baseline, repeated all seven Simulation 20.9 stress cases, and completed an independent $N = 2048$ fine-grid calculation. Every qualification case passed. The maximum compensated-balance residual improved from 1.79681×10^{-8} on the baseline grid to 4.17965×10^{-9} on the fine grid, while the baseline-to-fine field difference reproduced the convergence behavior independently measured in Simulation 20.8. Following review of the completed qualification matrix, Simulation 20.10 was locked as the qualified numerical solver.

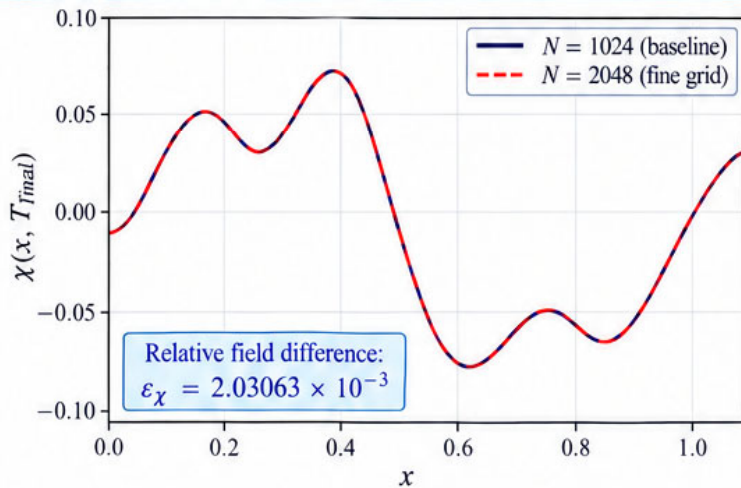
Figure 024. Simulation 20.10 — Final Integrated Solver Qualification

Independent baseline, complete stress matrix, and fine-grid verification for the LOCKED Simulation 20 solver.

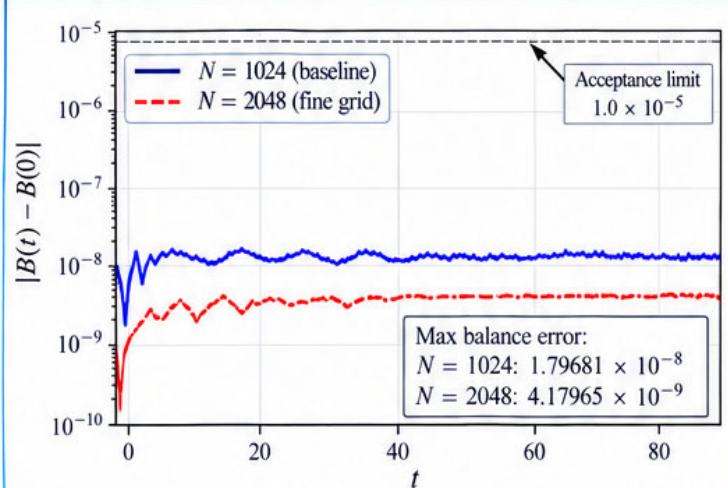
A. Qualification Matrix (Simulation 20.10)

Case	Description	Key Parameter Changes (from baseline)	$\max \chi $	Final Energy E_f	Max Balance Error	Result
Q0	Independent baseline ($N = 1024$)	Baseline coefficients	3.35313×10^{-2}	5.66518×10^{-4}	1.79681×10^{-8}	PASS
S0	Stress matrix control (baseline)	Baseline coefficients	3.35313×10^{-2}	5.66518×10^{-4}	1.79681×10^{-8}	PASS
S1	Cubic coefficient doubled	$\lambda_\chi = 40$	3.31954×10^{-2}	5.79478×10^{-4}	1.77739×10^{-8}	PASS
S2	Linear damping $\times 4$	$\gamma_\chi = 0.08$	2.50000×10^{-2}	2.45980×10^{-5}	2.98310×10^{-8}	PASS
S3	Nonlinear damping $\times 2$	$\eta_\chi = 160$	3.19329×10^{-2}	3.38165×10^{-4}	2.40977×10^{-8}	PASS
S4	Forcing amplitude $\times 4$	$F_\chi = 0.008$	5.86664×10^{-2}	2.65818×10^{-4}	4.19685×10^{-8}	PASS
S5	Forcing frequency change	$\omega_\chi = 0.47$	3.63410×10^{-2}	4.17970×10^{-4}	2.07730×10^{-8}	PASS
S6	Combined stress	$\lambda_\chi = 40, \eta_\chi = 160$ $F_\chi = 0.008, \omega_\chi = 0.47$	5.58222×10^{-2}	1.88822×10^{-4}	5.87265×10^{-8}	PASS
Q1	Independent fine grid ($N = 2048$)	Baseline coefficients	3.35471×10^{-2}	5.66420×10^{-4}	4.17965×10^{-9}	PASS

B. Baseline vs. Fine-Grid Field (Simulation 20.10)



C. Compensated Energy Balance (Simulation 20.10)



D. Simulation 20 Development Chain and Final Disposition

20.7B
PASS
(LOCKED)



20.8
PASS
(LOCKED)



20.9
PASS
(LOCKED)



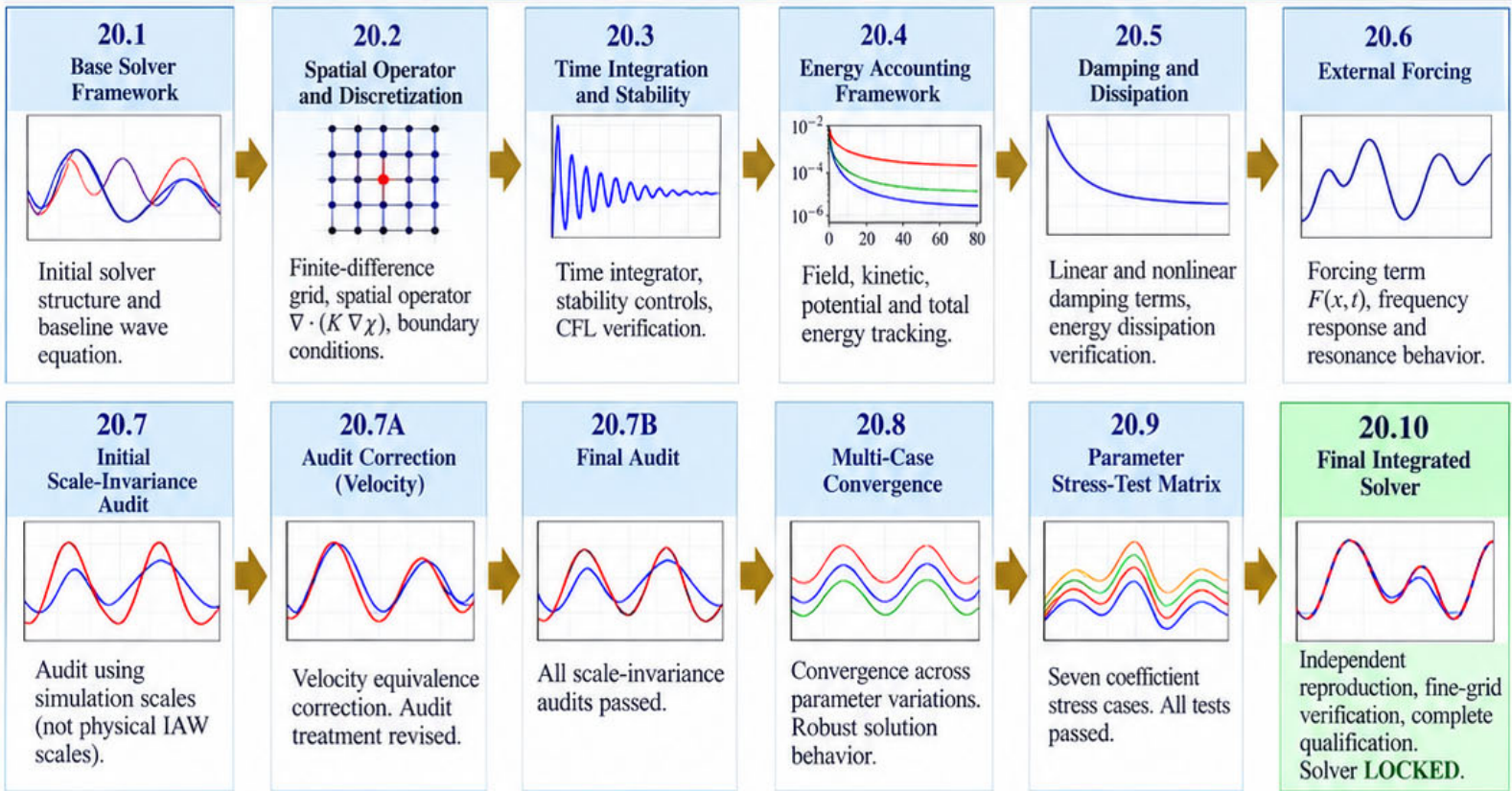
20.10
PASS
(LOCKED)

Key Conclusions (Simulation 20.10)

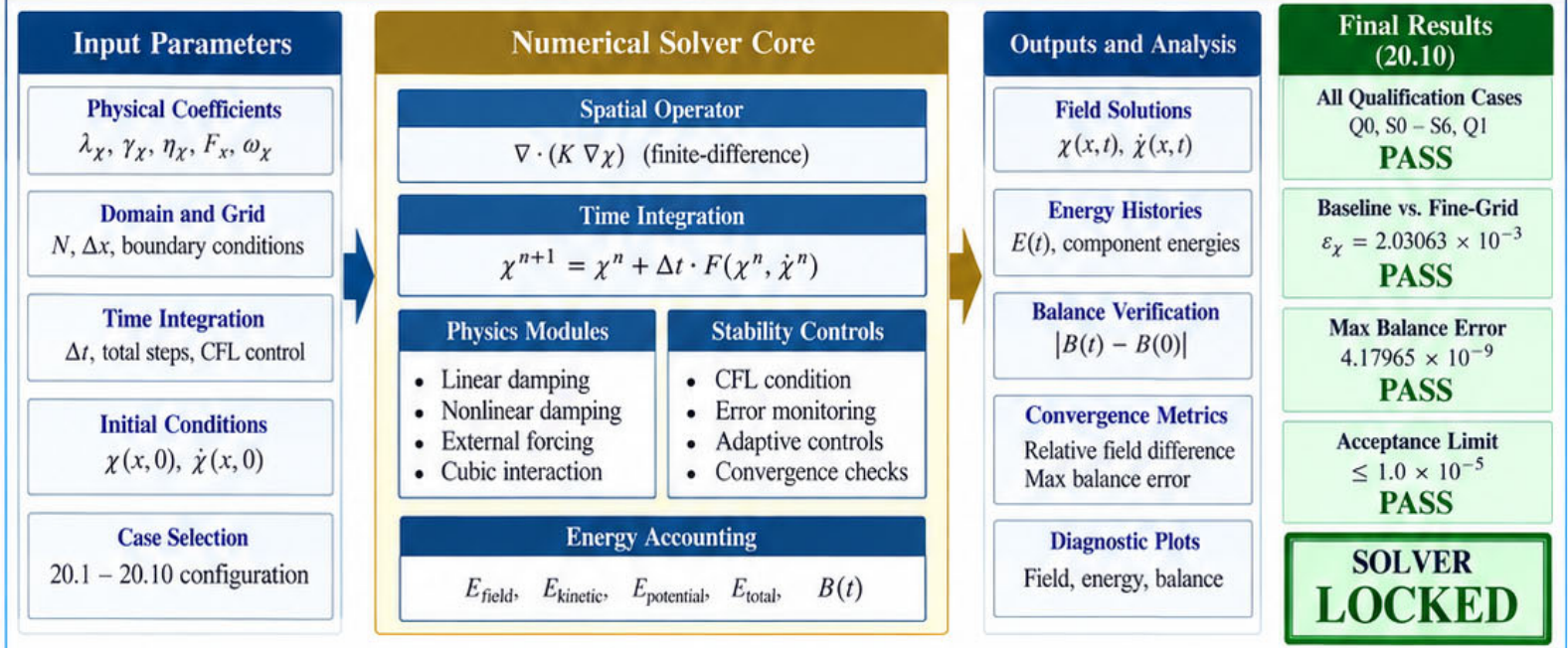
- Independent baseline reproduced (Q0: PASS).
- Complete stress matrix reproduced (S0–S6: PASS).
- Independent fine-grid calculation passed (Q1: PASS).
- Results consistent with earlier convergence (Simulation 20.8).
- All compensated-balance errors $\leq 1.0 \times 10^{-5}$ (largest: 5.87265×10^{-8}).
- Baseline-to-fine relative field difference: $\varepsilon_\chi = 2.03063 \times 10^{-3}$.
- Solver **LOCKED** as the qualified numerical engine.

Figure 025. Simulation 20.1 – 20.10: Solver Program Architecture

Development sequence, numerical components, and final LOCKED solver architecture.



Final Solver Architecture (Simulation 20.10)



Key Numerical Equation	Grid and Time Settings	Energy Definitions	Convergence Metrics
$\partial_t(\rho_{\text{eff}} \partial_t \chi) = \nabla \cdot (K \nabla \chi) + F(x, t) - \gamma_\chi \partial_t \chi - \eta_\chi (\partial_t \chi)^2 \partial_t \chi + \lambda_\chi \chi^3$	$N = 1024, 2048$ (verification) $\Delta x = 1.0 / (N - 1)$ Δt (CFL controlled) Finite-difference spatial operator Consistent boundary conditions	$E_{\text{field}} = \frac{1}{2} \int (\nabla \chi)^2 dx$ $E_{\text{kinetic}} = \frac{1}{2} \int (\partial_t \chi)^2 dx$ $E_{\text{potential}}$ (nonlinear terms) $E_{\text{total}} = E_{\text{field}} + E_{\text{kinetic}} + E_{\text{potential}}$ $B(t) = E_{\text{total}}(t) - E_{\text{total}}(0)$	Relative field difference: $\varepsilon_\chi = 2.03063 \times 10^{-3}$ Max balance error ($N = 1024$): 1.79681×10^{-8} Max balance error ($N = 2048$): 4.17965×10^{-9} Acceptance limit: 1.0×10^{-5}

